$n_{HOA,\ out}$