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Title:	Physical Limits of SU-MIMO Configurations for LTE-A
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1. Introduction

LTE-Advanced (LTE-A) targets an instantaneous downlink peak data rate of 1 Gbps [3]. In addition to an increased total bandwidth, this target can only be achieved by increasing the number of spatial multiplexed data streams towards a single-user (SU). The current assumption is to employ up to 8 antennas at both eNodeB and UE. In particular up to 4 antennas on a handset and 8 antennas on a laptop.
The theoretical capacity of a *uncorrelated* SU-MIMO channel is well known since the pioneering works of Telatar [1]. However, the antennas in practical devices cannot be spaced arbitrarily. Due to the physical dimensions the antennas tend to be correlated.
Given the surface of a transmitter and a receiver, it is difficult to say how many antennas can be employed before the mutual correlation cancels their benefits. Of course the number of antennas also depends on power consumption and other design aspects. In the following we will focus only on antenna correlation to determine a reasonable number of antennas. Commonly, it is acknowledged that any pair of antenna should be spaced at least by $\lambda/2$, but no strong theoretical argument supports this claim. Recent contributions [4],[5] allow to demystify some of the beliefs concerning MIMO spatial limitations. The following contribution

- analyzes the capacity gains engendered by $N \times N$ MIMO systems on limited surface devices
- shows that, even for a mobile phone terminal, the 8×8 MIMO setup provides significant capacity gain
- shows that, for large systems such as laptops, even 16×16 MIMO is theoretically interesting
- discusses the different problems practically encountered to reach such transmission rates.

The remainder of this contribution is organized as follows: Section 2 introduces and justifies the Kronecker channel model. Section 3 provides the formula for the capacity limit of *dense* MIMO setups which approximates the SU-MIMO capacity. Furthermore it provides a practical algorithm to compute the theoretical limit for a given transceiver design. Section 4 gives simulations results for various transceiver designs corroborating the theoretical claims. Section 5 discusses several key practical points to achieve high transmission rates. Finally, Section 6 states our conclusions.

2. System model, justification and usage

We consider a SU-MIMO downlink system with N_{T} transmit antennas at the eNodeB and N_{R} receive antennas at the UE. Both eNodeB and UE have a limited surface, which as a consequence entails correlation at the antenna arrays. The prior information on the communication channel can be summarized as: (i) the channel has a known mean power, (ii) the distances $d_{i,j}^{(\mathrm{T})}$ and $d_{i,j}^{(\mathrm{R})}$ between the transmit antennas (i,j) and the receive antennas (i,j) are known, respectively. This limited amount of knowledge implies that the Kronecker channel model is the most *appropriate* model given the prior information on the channel. This is demonstrated in [2]. Therefore, denoting $\mathbf{\Theta}_{\mathrm{T}}$ the correlation matrix at the transmitter, $\mathbf{\Theta}_{\mathrm{T}}$ the correlation matrix at the receiver and $\mathbf{H}_w$ some standard Gaussian i.i.d. distributed $N_{\mathrm{R}} \times N_{\mathrm{T}}$ complex matrix, the channel model $\mathbf{H}$ reads

$$\mathbf{H} = \mathbf{\Theta}_{\mathrm{R}}^{1/2} \mathbf{H}_w \mathbf{\Theta}_{\mathrm{T}}^{1/2} \tag{1}$$

Given the array of distances $\{d_{i,j}^{(\mathrm{R})}\}$ at the receiver for instance, we show, using the classical Jake's model[1] that $\mathbf{\Theta}_{\mathrm{R}}$ is of the form

$$\begin{bmatrix} 1 & J_0(\frac{2\pi}{\lambda} d_{1,2}^{(\mathrm{R})}) & J_0(\frac{2\pi}{\lambda} d_{1,3}^{(\mathrm{R})}) & \dots & J_0(\frac{2\pi}{\lambda} d_{1,N_{\mathrm{R}}}^{(\mathrm{R})}) \\ J_0(\frac{2\pi}{\lambda} d_{2,1}^{(\mathrm{R})}) & 1 & J_0(\frac{2\pi}{\lambda} d_{2,3}^{(\mathrm{R})}) & \dots & J_0(\frac{2\pi}{\lambda} d_{2,N_{\mathrm{R}}}^{(\mathrm{R})}) \\ J_0(\frac{2\pi}{\lambda} d_{3,1}^{(\mathrm{R})}) & J_0(\frac{2\pi}{\lambda} d_{3,2}^{(\mathrm{R})}) & 1 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ J_0(\frac{2\pi}{\lambda} d_{N_{\mathrm{R}},1}^{(\mathrm{R})}) & J_0(\frac{2\pi}{\lambda} d_{N_{\mathrm{R}},2}^{(\mathrm{R})}) & \dots & \dots & 1 \end{bmatrix} \tag{2}$$

In the following we consider the impact of correlation on the resulting achievable transmission rate. We especially treat the asymptotic case for which the number of antennas grows to infinity at both transmitter and receiver with a constant ratio $N_{\mathrm{T}}/N_{\mathrm{R}} = \beta$: this is referred to as the *dense* MIMO model or *dense* antenna model. This asymptotic study is interesting in the sense that the capacity achieved for an infinite number of antennas is already achieved for the "best" $N \times N$ scenario with N generally very small.

3. Physical limits of a MIMO system

We show in this section that the capacity achieved by the $N \times N$ MIMO setups saturates quickly when N grows to an quasi-optimal scenario. By quasi-optimal, we mean that adding more antennas does not contribute to a relevant increase in capacity. We also show that the peak rate corresponding to this quasi-optimal scenario can be always computed; for this, one needs to consider the asymptotic dense MIMO capacity which was recently shown to be easy to compute. A simple algorithm allows to assess exactly this capacity limit, which only depends on the shape and surface of both transmitter and receiver.

In a recent contribution [5] it is shown that the asymptotic (dense antenna) capacity of a correlated system only depends on the ratios "carrier frequency" over "system dimensions". For

[1] which is the least committal model given the prior assumptions on the antenna correlations

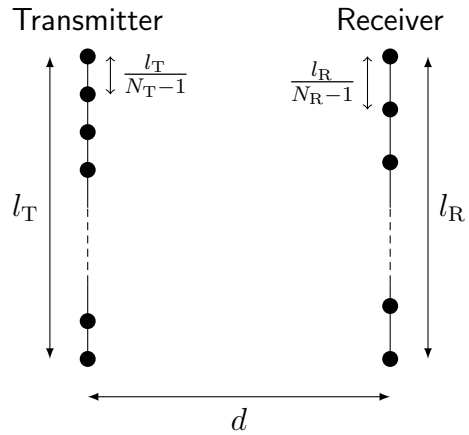

Figure 1: One-dimensional antenna array geometry

instance, consider first a setup of one-dimensional arrays as depicted in Figure 1. The transmitter is a line of length l_{T} while the receiver is a line of size l_{R}. In this case the correlation matrices entries are simple. For instance, we have

$$[\mathbf{\Theta}_{\mathrm{R}}]_{i,j} = J_0\left(\frac{2\pi}{\lambda} d^{\mathrm{R}}_{i,j}\right) = J_0\left(\frac{2\pi}{\lambda}\frac{|i-j|l_{\mathrm{R}}}{N_{\mathrm{R}}-1}\right) \tag{3}$$

Call $F_{\mathrm{R}}(k)$ the kth eigenvalue of $\mathbf{\Theta}_{\mathrm{R}}$, $k \in \{1, \ldots, N_{\mathrm{R}}\}$ and $F_{\mathrm{T}}(k)$ the kth eigenvalue of $\mathbf{\Theta}_{\mathrm{T}}$. Denote also $\rho' = \frac{\rho}{N_{\mathrm{R}}}$, the ratio between the total receive SNR ρ over the number of receive antennas N_{R}[2].

The general formulation for the ergodic capacity $\mathcal{C}$ of the asymptotic dense MIMO setup is shown to be a function of the parameters ρ', $F_{\mathrm{R}}(k)$ $(1 \leq k \leq N_{\mathrm{R}})$ and $F_{\mathrm{T}}(k)$ $(1 \leq k \leq N_{\mathrm{T}})$ [5]. The formula is provided in the appendix.

This result allows to find the dense MIMO capacity via a fixed point algorithm on the variables Γ and Υ, which we also describe in the appendix.

Simulations indicate that the theoretical capacity limit is slightly lower compared to the optimal capacity, when the antennas are uniformly distributed on the available surface; this is due to minor edge effects. However, the approximation is fairly accurate and allows to easily compute the capacity saturation for large MIMO setups without Monte-Carlo simulations over many channel realizations.

Note that the limit capacity only depends on the eigenvalue distribution of the correlation matrices $\mathbf{\Theta}_{\mathrm{R}}$ and $\mathbf{\Theta}_{\mathrm{T}}$. Therefore, we can intuitively understand that, by densifying the antennas, most distances $d^{(\mathrm{R})}_{i,j}$ and $d^{(\mathrm{T})}_{i,j}$ tend to be equal, and by continuity of the Bessel J_0 function, most entries of $\mathbf{\Theta}_{\mathrm{R}}$ and $\mathbf{\Theta}_{\mathrm{T}}$ tend to be equal. Therefore, the effective ranks of those matrices tend to saturate. This means that there should exist a limit scenario $N_{\mathrm{sat}} \times N_{\mathrm{sat}}$ such that, for any denser MIMO setup, the capacity does not increase. As a consequence, the aforementioned

[2]this is required to simulate the size reduction of every antenna when increasing their number on the fixed receiver surface.

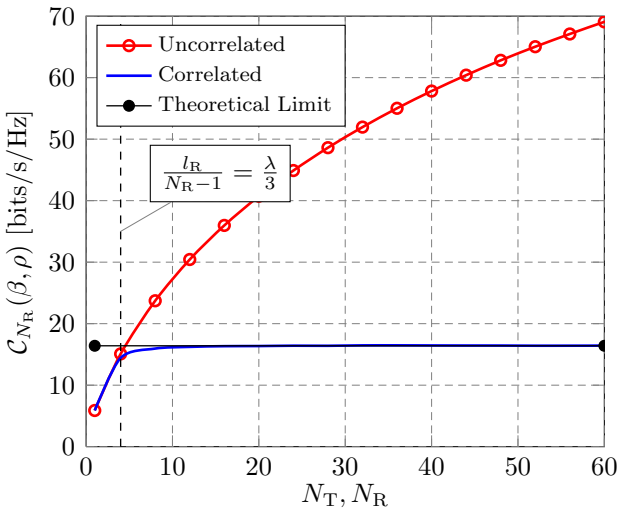

Figure 2: **One-dimensional setup**, ergodic MIMO capacity $\mathcal{C}_{N_R}(\beta, \rho)$, $N_R = N_T$, $\rho' = 20$ dB

asymptotic capacity limit corresponds to the capacity achieved by this $N_{sat} \times N_{sat}$ setup. This is proved by simulations in the following.

4. Simulation and results

In the following, we consider a one-dimensional setup and a two-dimensional setup.

4.1. One-dimensional setup

Consider a line of antennas of length $l_R = 10$ cm at the UE and length $l_T = 3$ m at the eNodeB. The signal wavelength is $\lambda = 10$ cm corresponding to a carrier frequency of 3 GHz. The antennas are uniformly distributed on the linear arrays. The results are depicted in Figure 2. As expected we observe a saturation of the capacity when increasing the number of transmit and receive antennas. The saturation level is shown to be well described by the theoretical limit of the dense MIMO setup. Remark also that, as previously stated, the 4×4 MIMO scenario almost corresponds to the saturated capacity. This means that no more than 4 antennas would be needed in this one-dimensional scenario; this also shows that the asymptotic capacity, which can be easily assessed, is a fairly good approximation of this "best" 4×4 setup. Note also that in this case $l/\lambda = 1$ is non negligible and proves to be detrimental to the capacity. Hence, the achieved capacity is upper-bounded because of the size of the UE.
Indeed, with $N_R = 4$, the capacity scales linearly with $\min(N_R, N_T)$ as in the case of uncorrelated antennas. This scenario corresponds to a minimal distance of $\lambda/3$ between the antennas at the UE.

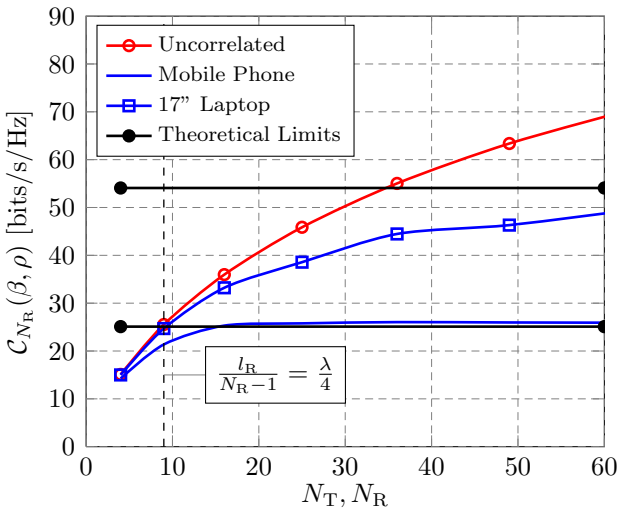

Figure 3: **Two-dimensional setup**, ergodic MIMO capacity $\mathcal{C}_{N_R}(\beta,\rho)$, $N_R = N_T$, $\rho' = 20$ dB

4.2. Two-dimensional setup

Consider now a square surface at the eNodeB of size $1\,\mathrm{m} \times 1\,\mathrm{m}$ with increasing number of antennas and a rectangular UE with dimensions $10\,\mathrm{cm} \times 5\,\mathrm{cm}$ (mobile phone) or $40\,\mathrm{cm} \times 30\,\mathrm{cm}$ (17" laptop). All antennas are uniformly distributed on a grid. The performance in terms of capacity is depicted in Figure 3. The surprising result here is that the capacity scales like the ideal uncorrelated case up to $N_R \simeq 9$ for the mobile phone. This corresponds to the case of a 3×3 grid of antennas, spaced by a minimal distance of $\lambda/4$. Note also that the laptop UE capacity can grow very large and almost fits the uncorrelated scenario when $N_R \leq 16$. This suggests that even higher MIMO configurations than 8×8 are theoretically possible.

5. Discussion

As presented previously, and thoroughly detailed in [5], we observe in simulation that the dense MIMO limit is a good approximation of the optimal MIMO capacity for a uniform distribution of the antennas.

We verified that the 4×4 and even 8×8 MIMO scenarios still achieve a non-negligible capacity growth for both a laptop and a mobile phone. Indeed, in case of a laptop the capacity grows linearly up to a 16×16 MIMO configuration.

However, it is important to consider those results carefully. The system model assumes a rich scattering environment[3]. In channels with higher correlation this model is not applicable. The presented results hold only for rich scattering scenarios. Nevertheless, we expect those results to be very close to the best case scenarios that can be encounter in rich scattering environments.

[3]expressed by the inner Gaussian matrix $\mathbf{H}_w$

This study does not provide an insight on the practical feasibility on such embedded scenarios. Especially for small devices such as a mobile phone terminal there are different challenges:

- Dimensionality: how to place up to 8 antennas and their associated analog components in a small device?
- Power consumption: even if the total transmitted power is the same on an array of N antennas or on a single antenna, the signal processing effort would raise the power consumption.
- Correlation: this study assumes the ideal case in which only distances between antennas affect the correlation. In practice, all the electronic components severely affect the correlation factor, as well as the human interaction (e.g. placing the hand round the phone).

It is therefore difficult to estimate the relevance of this study to practical cases.
Note also that we did not evaluate the performance of cross-polarized antennas neither at the transmitter or the receiver. Additional antennas can fit in the previous design if a proper correlation model for cross-polarized antennas is added to this study.

6. Conclusion

This contribution provides physical limits of the capacity between an eNodeB and a UE of limited dimensions. We showed in particular that the 4×4 and 8×8 MIMO configurations targeted for LTE-A are reasonable from a theoretical point of view, even for communications with a mobile phone terminal. We also provided a tool which allows to compute an upper-bound on the theoretical capacity that can be achieved limited antenna scenarios.

A. Dense MIMO capacity

The capacity of the dense $N_\mathrm{R} \times N_\mathrm{T}$ MIMO system is given by

$$\begin{aligned}\mathcal{C}(\beta, \rho') = &\sum_{k=0}^{N_\mathrm{T}} \log(1 + \beta\rho' F_\mathrm{T}(k)\Gamma) \\ &+ \sum_{k=0}^{N_\mathrm{R}} \log(1 + \rho' F_\mathrm{R}(k)\Upsilon) \\ &- \beta \cdot \rho' \cdot \Gamma(\rho')\Upsilon(\rho') \log(e)\end{aligned} \tag{4}$$

with

$$\Gamma(\rho') = \frac{1}{\beta N_\mathrm{R}} \sum_{k=0}^{N_\mathrm{R}} \frac{N_\mathrm{R} F_\mathrm{R}(k)}{1 + \rho' F(k)\Upsilon} \tag{5}$$

$$\Upsilon(\rho') = \frac{1}{N_\mathrm{T}} \sum_{k=0}^{N_\mathrm{T}} \frac{N_\mathrm{T} F_\mathrm{T}(k)}{1 + \rho'\beta F(k)\Gamma}. \tag{6}$$

B. Algorithm

The necessary steps to compute the previously defined capacity dense MIMO $\mathcal{C}$ can be summarized as follows:

1. For a given system, design the distance relations $d^{(\mathrm{R})}$ and $d^{(\mathrm{T})}$ that must scale with N_R and N_T.
2. Generate the corresponding correlation matrices $\mathbf{\Theta}_\mathrm{R}$ and $\mathbf{\Theta}_\mathrm{T}$.
3. For large[4] N_R and N_T and $N_\mathrm{T}/N_\mathrm{R} = \beta$, compute the eigenvalues $F_\mathrm{R}(k)$, $F_\mathrm{T}(k)$ of $\mathbf{\Theta}_\mathrm{R}$, $\mathbf{\Theta}_\mathrm{T}$, respectively.
4. *Initialization*: Set $\Gamma = 0$, $\Upsilon = 0$.
5. *Loop*: Iteratively compute Γ and Υ according to equations (5) and (6).
6. *Termination*: After a sufficient number of iterations, compute the capacity $\mathcal{C}$ according to formula (4).

[4] e.g. of order 256 to 1024

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