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Title: **Link-level results for nonunitary precoding using zero-forcing in LOS channels**
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1. Introduction

We propose that nonunitary precoding be considered as an option for MU-MIMO transmission. Specifically, and in order to illustrate the potential of nonunitary precoding, we consider zero-forcing (ZF) precoding in which users experience no interbeam interference when the channel state information (CSI) is known exactly at the NodeB. CSI at the NodeB transmitter has been shown to improve the performance of downlink single-user and multiuser MIMO techniques [1] under the assumption of ideal CSI knowledge. In this contribution, we study the performance of ZF under non-ideal CSI as a result of limited uplink feedback. We compare its link-level performance in line-of-sight channels with conventional (PU2RC) beamforming by parameterizing the number of feedback bits per uplink channel use.

2. Description of Zero-Forcing Beamforming

We give a high-level description of nonunitary precoding based on zero-forcing (ZF) in this section. A more detailed mathematical description is given in Appendix A. As shown in Figure 1, we assume that the NodeB uses *M* antennas to simultaneously serve a set of single-antenna users denoted by *S*. We let **x**(*S*) denote the |*S*|-dimensional vector of transmitted signal vectors (where |*S*| denotes the cardinality of *S*). We let **G**(*S*) denote the *M* -by- |*S*| matrix of precoding (beamforming) weights.

In conventional beamforming, the columns of **G**(*S*) correspond to the precoding vectors indexed by the UE uplink feedback. These vectors typically correspond to columns of unitary matrices. With *B* bits of feedback per UE, the size of the codebook is 2^B vectors. ZF must also rely on quantized uplink feedback from the UE for determining the beamforming weights. However, unlike conventional techniques where the NodeB simply uses the beamforming vectors requested by the UE, ZF uses the requested vectors to calculate a new set of weight vectors that reduces the interbeam interference. This calculation is performed on each TTI and on each resource block based on the UE feedback.

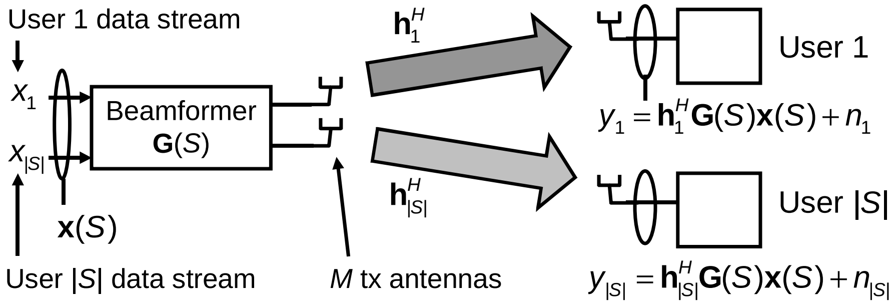

Figure 1. Block diagram for transmitting to a set of users *S* using precoding matrix **G**(*S*).

Under ideal conditions, in which the UE could feedback the MISO channel coefficients exactly without quantization, the ZF beamformer would be computed from the exact MISO channels, and it would be ideal in the sense that users would experience no interbeam interference. In other words, the received signal by the *k*th user (*k* = 1,…,|*S*|) would be $y_k = x_k + n_k$. We note that, for conventional beamforming, even under ideal feedback conditions where the transmitter knew the channel exactly (a codebook with infinitely many precoding vectors), there would be interbeam interference. We show the performance of the ideal ZF beamformer and compare it to the capacity-achieving dirty paper coding technique in Appendix B.

With quantized feedback, the ZF beamformer is no longer ideal because each user's received signal will be corrupted by interbeam interference from other users' signals. The conventional beamformer will likewise become more coarse with quantized feedback. Despite the degradation of the ZF performance due to quantized feedback, we will show that it potentially provides significant gains over conventional PU2RC beamforming for the same level of quantization. The goal of this contribution is to quantify these performance gains for line-of-sight channels.

3. Simulation results

We now consider the ZF performance with limited feedback. We assume that the NodeB uses a linear array with $M = 4$ antenna elements with 0.5 wavelength spacing. We assume a line-of-sight (LOS) channel model so that, given a user with SNR s and radial direction θ with respect to the antenna array boresight direction, its MISO channel is

$$\mathbf{h}_k = \sqrt{s}\begin{bmatrix} 1 \\ \exp[-j\pi\sin(\theta)] \\ \vdots \\ \exp[-j\pi(M-1)\sin(\theta)] \end{bmatrix}$$

The codebooks for PU2RC and the vectors for generating the ZF beamforming vectors are based on the same set of vectors $\mathbf{w}_b$, $b = 1,\ldots,2^B$ [2], where the mth element of $\mathbf{w}_b$ is $\frac{1}{\sqrt{M}}\exp\left(\frac{j2\pi(m-1)(b-1)}{2^B}\right)$. The simulation parameters are summarized in Table 1.

Number of transmit antennas	$M = 4$
Element spacing	0.5 wavelength
Number of antennas per UE	$N = 1$
Beamforming techniques	Conventional (PU2RC) and Zero-forcing
Channel model	Line-of-sight
Modulation and coding schemes	QPSK, 16QAM, and 64QAM, Rel-6 turbo codes, rates 0.1, 0.14, 0.2, 0.25, 0.33, 0.4, 0.5, 0.6, 0.67, 0.75, 0.8, 0.89
FER	1%
Channel feedback bits	$B = 3, 5, 7$
UEs per cell	$K = 5, 10, 20, 40$

Table 1: Link-level simulation assumptions.

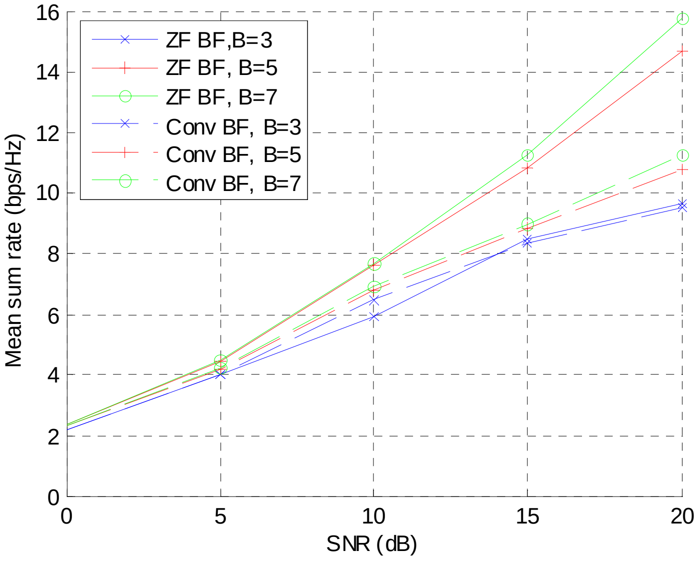

Figure 2. Link-level performance of zero-forcing and conventional (PU2RC) beamforming, parameterized by the number of channel feedback bits B, $K = 20$ users.

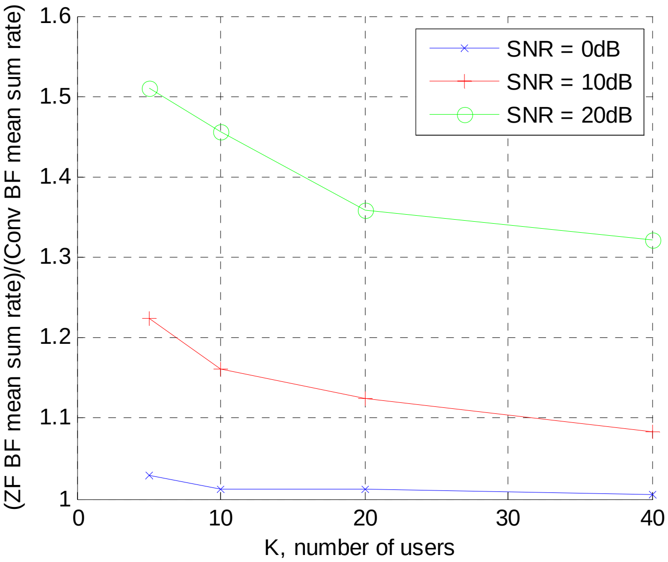

Figure 3. Performance gain of zero-forcing BF over conventional BF, parameterized by the average SNR. Number of feedback bits *B* = 5.

Figure 2 shows the mean sum rate (averaged over random angular placements of users) versus the users' average SNR*s*. For *B* = 5 or 7 bits, ZF beamforming provides gains over conventional beamforming over the entire range of SNRs. The performance gains are quite significant at high SNRs. We note that there is not significant improvement in performance in going from 5 to 7 bits for both ZF and conventional beamforming.

Figure 3 gives the ratio of the mean sum rates of ZF and conventional beamforming for *B* = 5 bits. For high SNRs (SNR = 20dB), ZF provides over 30% gain using the same feedback resources as conventional beamforming.

4. Conclusions

We presented preliminary link-level simulation results for a nonunitary precoding technique based on zero-forcing beamforming. This technique uses the same uplink feedback for conventional beamforming but computes a set of zero-forcing beamforming weights that reduces inter-beam interference. The performance gains of ZF are significant for high SNRs and small numbers of users. These gains are attractive because they are the result of a minimal additional complexity. Future work should address the performance in non-LOS channels and the impact of receiver channel estimation and CQI feedback delays.

5. References

[1] R1-061877, Lucent Technologies, "Downlink enhancements using additional antennas, full channel knowledge, and multiuser eigenmode transmission".

[2] R1-060335, Samsung, "Downlink MIMO for EUTRA".

[3] G. Dimic and N. D. Sidiropoulos, “On downlink beamforming with greedy user selection: Performance analysis and a simple new algorithm,” *IEEE Trans. Commun.*, vol. 53, no. 10, pp. 3857-3868, Oct. 2005.

Appendix A: Detailed description of beamforming techniques

We assume that the NodeB has M transmit antennas and a codebook consisting of 2^B precoding (beam) vectors, each designated by a normalized M-dimensional complex vector $\mathbf{w}_b$, $b = 1,\ldots,2^B$, with $\|\mathbf{w}_b\| = 1$. Each UE feeds back B bits to index one of the precoding vectors. Let K be the number of users per cell, and for a given resource block, let $\mathbf{h}_k$ be the flat-fading M-dimensional MISO channel vector at the kth user, $k = 1,\ldots,K$. For a channel realization $\mathbf{h}_k$, the kth user chooses the vector according to the minimum distance criterion:

$$b_k = \underset{b=1,\ldots,B}{\arg} \max |\mathbf{h}_k^H \mathbf{w}_b| / \|\mathbf{h}_k\| .$$

The index of the chosen beam vector is fed back to the NodeB. Out of the K total users, the NodeB must select a subset of users to serve in order to maximize some performance metric, for example, the sum rate. We let S^* denote this optimum subset and let $|S^*|$ denote its cardinality. The transmitter block diagram is shown in Figure 4 for a general set S, where $\mathbf{x}(S)$ is the $|S|$-dimensional vector of transmitted symbols and $\mathbf{G}(S)$ is the M-by-$|S|$ beamforming matrix. The jth element of $\mathbf{x}(S)$ is the data symbol for the jth user, $j = 1,\ldots,|S^*|$. (For simplicity of notation, we assume the users are renumbered to correspond to set S.) The optimum set S^* is the one chosen among all possible sets S which maximizes the sum rate.

We now describe how this set S^* is chosen under zero-forcing beamforming. For a given subset of users S, the zero-forcing beamforming matrix is given by the M-by-$|S|$ matrix

$$\mathbf{G}(S) = \mathbf{W}(S)\left[\mathbf{W}^H(S)\mathbf{W}(S)\right]^{-1}, \tag{1}$$

where $\mathbf{W}(S)$ is the M-by-$|S|$ matrix of whose columns correspond to the beam vectors of the users in set S. Specifically, the jth column ($j = 1,\ldots,|S|$) of $\mathbf{W}(S)$ is $|\mathbf{h}_{k(j)}|\mathbf{w}_{b_{k(j)}}$, where $k(j)$ is the user index corresponding to the jth element of S and where the magnitude of $\mathbf{h}_{k(j)}$ is obtained from the CQI feedback. Note that a necessary condition for the matrix inverse to exist is $|S| \le M$. In the remainder of this contribution, we simplify the notation by assuming each user in set S is renumbered to correspond to its order in the set. In other words, the jth column ($j = 1,\ldots,|S|$) of $\mathbf{W}(S)$ would be written as $|\mathbf{h}_j|\mathbf{w}_{b_j}$. Therefore the maximum number of users that can be simultaneously served under ZF is M. The transmitted signal is $\mathbf{G}(S)\mathbf{x}$, and the received signal by the users in S can be written as an $|S|$-dimensional vector:

$$\mathbf{y} = \mathbf{H}^H(S)\mathbf{G}(S)\mathbf{x}(S) + \mathbf{n} \tag{2}$$

where $\mathbf{H}(S)$ is the M-by-$|S|$ channel matrix and the elements of the additive noise vector $\mathbf{n}$ are IID zero-mean complex Gaussian random variables with unit variance. Because the NodeB does not have knowledge of $\mathbf{H}(S)$, it assumes the channel matrix is given by $\mathbf{W}(S)$ in computing the weighted sum rate. Replacing $\mathbf{W}(S)$ with $\mathbf{H}(S)$ in (1) yields a received signal by each user with additive noise but no interuser interference: $\mathbf{y} = \mathbf{x}(S) + \mathbf{n}$. This is the desired effect of the zero-forcing beamforming. The sum rate for a set S is given by maximizing

$$\sum_{k \in S} \log_2\left(1 + E\left[\|x_k\|^2\right]\right), \tag{3}$$

over the transmit power for each user, subject to a sum transmit power constraint over the M antennas $tr\left\{E\left[\mathbf{G}(S)\mathbf{x}(S)\mathbf{x}^H(S)\mathbf{G}^H(S)\right]\right\} \le P$. The optimum transmit powers can be found via waterfilling. Later in performing the link-level simulations, we will assume that all users have the same average SNR. Under this assumption, it is reasonable to further assume equal transmit power for each user (in other words, $E\left[\|x_k\|^2\right] = E\left[\|x_j\|^2\right]$ for all $j,k \in S$). If $P = 1$, the optimum transmit power per user is

$$\bar{v} = \left(\sum_{k \in S} \sum_{m=1}^{M} \left| \mathbf{G}_{m,k}(S) \right|^2 \right)^{-1} . \tag{4}$$

Under this condition of equal transmit powers, the optimum set of users *S** to serve is the one that maximizes the sum rate over all sets with cardinality less than or equal to *M*:

$$S^* = \underset{S,|S| \le M}{\arg\max} \left\{ |S| \log_2 \left[1 + \left(\sum_{k \in S} \sum_{m=1}^{M} \left| \mathbf{G}_{m,k}(S) \right|^2 \right)^{-1} \right] \right\} . \tag{5}$$

The optimum set is found by performing a brute-force search over all possible sets *S*. The number of sets to consider is $\sum_{j=1}^{\min(M,K)} \frac{K!}{(K-j)!\,j!}$, so the complexity of this search may be overwhelming if *K* is large. For this reason, a suboptimum greedy algorithm [3] can be used for determining the set *S**.

Once the optimum set S^* is chosen, the sum rate must be computed using the actual channel matrix $\mathbf{H}(S^*)$ in place of the assumed channel $\mathbf{W}(S^*)$. Note that unless $\mathbf{W}(S^*) = \mathbf{H}(S^*)$, each user will receive residual interference because the zero-forcing beamforming will not be ideal. The received signal by the *k*th user is

$$\begin{aligned} y_k &= \mathbf{h}_k^H \mathbf{G}(S^*) \mathbf{x}(S^*) + n_k \\ &= \mathbf{h}_k^H \mathbf{g}_k x_k + \sum_{j \in \bar{S},\, j \ne k} \mathbf{h}_k^H \mathbf{g}_j x_j + n_k . \end{aligned}$$

The sum rate is given by

$$\sum_{k \in S^*} \log(1 + \gamma_k) , \tag{6}$$

where $\gamma_k = \frac{\bar{v} \left| \mathbf{h}_k^H \mathbf{g}_k \right|^2}{\bar{v} \sum_{j \in S^*, j \ne k} \left| \mathbf{h}_k^H \mathbf{g}_j \right|^2 + 1}$ is the SINR for user *k* and $\bar{v} = \left(\sum_{k \in S^*} \sum_{m=1}^{M} \left| \mathbf{G}_{m,k}(S^*) \right|^2 \right)^{-1}$ is the transmit power for each user.

The operation and performance of conventional beamforming can be derived in a similar manner by replacing the ZF beamforming matrix in (1) with a conventional beamformer matched to the estimated MISO channels. In other words, for a given candidate set of users *S*, the beamforming matrix is $\mathbf{G}(S) = \mathbf{W}(S)$. The estimated SINR for the *k*th user computed at the NodeB is $\frac{\bar{v} \left| \mathbf{w}_{b_k} \right|^4}{\bar{v} \sum_{j \in S, j \ne k} \left| \mathbf{w}_{b_k}^H \mathbf{w}_{b_j} \right|^2 + 1}$, where the transmit power per user is $\bar{v} = \left(\sum_{k \in S} \sum_{m=1}^{M} \left| \mathbf{W}_{m,k}(S) \right|^2 \right)^{-1}$ is given by , and the optimum set *S** is

$$S^* = \underset{S,|S| \le M}{\arg\max} \sum_{k \in S} \log_2 \left[1 + \frac{\bar{v} \left| \mathbf{w}_{b_k} \right|^4}{\bar{v} \sum_{j \in S, j \ne k} \left| \mathbf{w}_{b_k}^H \mathbf{w}_{b_j} \right|^2 + 1} \right] . \tag{7}$$

The actual achieved sum rate is given by (6), using *S** from (7) and $\mathbf{G}(S^*) = \mathbf{W}(S^*)$.

Appendix B: Theoretical simulated performance of ideal ZF

To further motivate the use of ZF beamforming, we compare its performance with the capacity-achieving dirty paper coding technique. Figure 4 shows the average sum rate link-level performance for the two techniques when *K* = 4 or *K* = 20 users are served by a single NodeB with *M* = 4 antennas. The averaging is performed over IID Rayleigh-faded channel realizations and the CSI is assumed to be known perfectly at the transmitter. Under these conditions, the performance of ZF is within 2dB of the optimum DPC performance. As *K* increases, the performance gap narrows because it is more likely to find a set of *M* users whose channels are increasingly mutually orthogonal.

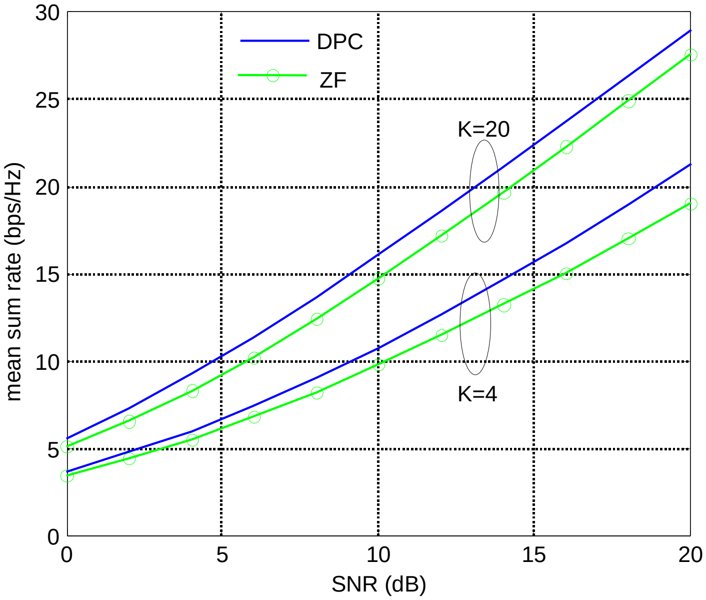

Figure 4. Average sum rate of capacity-achieving DPC and Zero-forcing beamforming.