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Agenda Item: Adhoc 5
Source: SAMSUNG Electronics Co.
Title: Results of Hamming weight asymptote and performance analysis for Turbo interleaver candidates
Document for: Discussion

1. Introduction

In this document, we analyze the relatively low free distance of Turbo codes resulting from weight-one information sequence and apply two different modifications to improve "Hamming weight asymptote (HWA)" proposed by NTT DoCoMo to obtain more reliable performance bound[1]. The motivation of this work is based on the fact that all inteleaver candidates might not consider effect of weight-one information sequence seriously. As a result, some interleavers have shown the error floor. In this paper, we propose that weight-one information sequence should also be considered as a critical pattern to design Turbo interleaver with a finite input block size.

2. Improved Hamming weight asymptote

The performance of Turbo codes is verified by examining the code's distance spectrum. It is well known that the "error floor" that occurs at moderate signal-to-noise ratios is a consequence of the relatively low free distance of the code. In the previous discussion of AdHoc5, "Hamming weight asymptote (HWA)" for the evaluation of the error-floor performance instead of BER simulation was proposed by NTT DoCoMo[1], [2] and some HWA results have shown consistency with the simulated performance of Turbo codes[1]. However, in [1] HWA is obtained only considering weight-two input sequence which results in low Hamming weight in generated code words[1],[3]. Also, in the last meeting, Lucent Technologies provided minimum weight comparison of four different classes of interleavers that have been proposed as candidates for 3GPP standardization[4]. In [4] and [5], it is found that some interleavers have very low Hamming weight of the generated code words and the corresponding information weight is one. In the TABLE1 some examples are given. In the following, we investigate the reason of this problem .

TABLE 1. FREE DISTANCE of 4 CANDIDATE INTERLEAVERS

Interleaver Size	S-random (Motorola)	PIL (NTT DoCoMo)	Prunable (HNS)	OCPNI (SEC)
478	17/1/1		17/1/1	
640		21/1/1		
720				21/1/1

- A/B/C means that A is the true free distance of the code, B is the number of code words with weight equal to the free distance, and C is the Hamming weight of the error pattern corresponding to the error events at free distance[4].

In Fig.1, an example of Turbo interelaving rule is given where F:{x} $\rightarrow$ {y} presents an interleaving rule, x(I), i=0,1,2,..,N-1 is the input information symbol for the first RSC encoder RSC1, and y(I), i=0,1,2,..,N-1 is the interleaved information symbol for the second RSC encoder RSC2, respectively. Assume that all the input information symbols are zero except the last one x(N-1) and F maps x(N-1) to y(n-1). Then, it is easily verified that Hamming weight of information symbol $W_H(x)=1$ and Hamming weight of redundancy of RSC1 corresponding to this x(0)..x(N-1) is exactly one. Furthermore, Hamming weight of redundancy of RSC2 is also exactly one! So, surprisingly the total Hamming weight of output code words $W_H(y)$ is only 3. Although this example is an extreme case, this implies the importance of handling the last input information symbol and its neighbors.

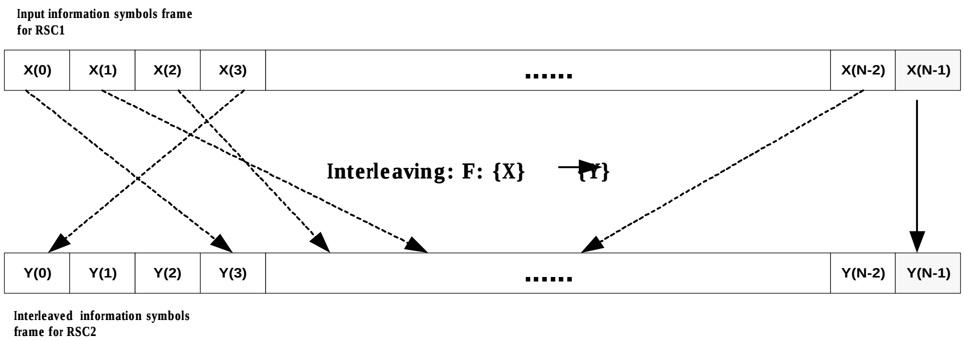

Fig.1. An example of the worst case of Turbo interleaving for a information weight of 1.

In Fig.2, another example is shown where an input information sequence with weight of 2 is considered. Note that this input information pattern is not like those patterns $1+D^j$, j=7,14,21,28 (critical information pattern) used in [1] where input length 8, 15, 22, 29 are considered. According to the previous analysis, it is clear that a low Hamming weight code word will occur although the input information sequence is not $1+D^j$, j=7,14,21,28,…So, it is also required to design an iterleaver avoiding these patterns.

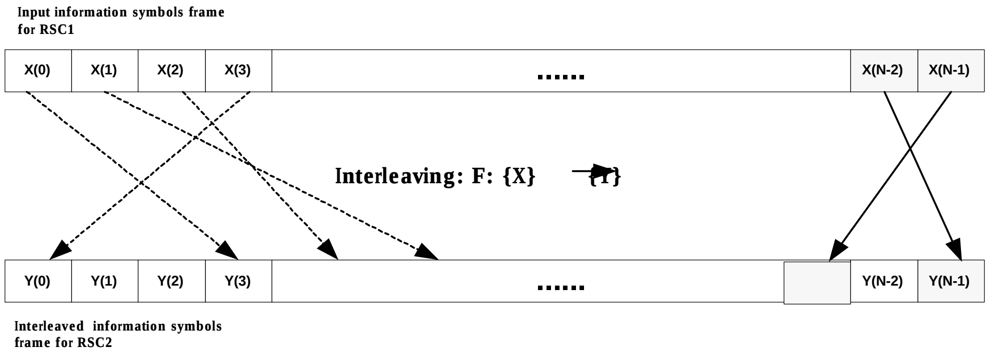

Fig.2. An example of worse case of Turbo interleaving for a information weight of 2 .

In conventional approach to design a good Turbo interleaver, input information sequences with weight of 2 are considered because they produce a critical pattern reducing Hamming weight of output code words. In fact, those input information sequences make minimum free distance of most of Turbo codes. Thus, it should be treated as the first design criterion. However, if an input sequence is truncated into some fixed block size then we should also consider an information sequence with weight of one, which is at the last position or near the last position of input frame. For the case of Fig.1, a simple method to prevent this low Hamming weight of output sequence is to move the last input symbol to the front of input frame or some position as far as an interleaving rule F can provides. For the case of Fig.2, a simple method to prevent that problem is similar to the previous one.

By following the previous analysis of Turbo codes, we modify Hamming weight asymptote (HWA) given in [1] to improve reliability of HWA performance bound. HWA is used for the evaluation of the error-floor performance instead of BER simulation due to too much simulation time to obtain reliable BER performance of all the frame sizes from 320 to 8129 bits at the moderate and high E_b/N_0 region (error-floor region). In reference [1] and [2], the union bound (an upper bound of BER) P_b of a N-bit-length turbo code with the maximum-likelihood (ML) decoding on additive white Gaussian noise (AWGN) channel is expressed by

$$P_b = \sum_{d=d_{free}}^{2(n+N)} \frac{N_d \, \tilde{w}_d}{N} Q\left(\sqrt{d\,\frac{2RE_b}{N_o}}\right)\Bigg|_{\tilde{w}_d=2-8} \tag{1}$$

where d_{free} is the free distance of the code, w_d is the average information weight per code word, N_d is the number of code words of weight d, R is code rate (in [1] R=1/2 is used), and n is the number of tail bits. Note that this probability only considered information sequence with weight of 2-8. In order to reflect the effect the information sequence with weight one, equation (1) should be modified as follows.

$$P_b = \sum_{d=d_{free}}^{3(n+N)} \frac{N_d \, \tilde{w}_d}{N} Q\left(\sqrt{d\,\frac{2RE_b}{N_o}}\right)\Bigg|_{\tilde{w}_d=2-8} + \sum_{d=d_{free}}^{3(n+N)} \frac{N_d \, \tilde{w}_d}{N} Q\left(\sqrt{d\,\frac{2RE_b}{N_o}}\right)\Bigg|_{\tilde{w}_d=1} \tag{2}$$

In equation (2), code rate R of 1/3 is used since the total code rate of PCCC encoder for 3GPP is 1/3. If the first term is not greater than the second term or at least the second term is close to the first term, then difference in performance between equation (1) and (2) becomes larger. Note that in the first term the events described in Fig.2 should also be considered for possible interleavers.

3. Hamming weight asymptote and BER performance of PIL

The minimum free distance of Turbo codes using PIL[6] for some specific frame sizes of 640 to 8192 bits given in Table 2 were calculated and shown in Table 3. All interleaver sizes in Table 2 are selected from the interleaver sizes corresponding to the case B of the second stage of PIL interleaving algorithm as in the following.

Second Stage:

B. If C = p+1

(B-1) Same as case A-1.
(B-2) Same as case A-2.
(B-3) Same as case A-3.
(B-4) Same as case A-4.
(B-5) Perform the j-th (j = 0,1, 2, …, R-1) intra-row permutation as:

$c_j(i) = c(\lfloor i \times p_j \rfloor \bmod (p-1))$, i =0,1,2, .., (p-2)., $c_j(p$-1) = 0, and $c_j(p) = p$,
where $c_j(i)$ is the input bit position of i-th output after the permutation of j-th row.

In (B-5), the last step "$c_j(p) = p$" makes the weight one problem mentioned in the previous chapter. Thus, it makes the very low free distance and should be discarded through some modification of PIL algorithm. In Table 3, the minimum free distances of the original PIL interleavers are given. Note that the minimum free distance remaines constant regardless of interleaver sizes. For N=8040, the minimum free distance of PIL is only 21. This result implies that the error pattern with Hamming weight of one causes the problem in PIL. In order to reduce this possibility of low minimum free distance, we propose two kinds of modifications as follows. It may also be possible to use other modifications.

B.1 PILSS (Method 1)

(B-6) exchange $c_{R-1}(p)$ with $c_{R-1}(0)$

B.2 PILSS2 (Method 2)

(B-6) exchange $c_{R-1}(p)$ with $c_{R-1}(p-1)$

Note that it is sufficient to modify the last step only in order to solve the problem and improve performance of PIL interleaver!

In Table 3 the minimum free distances of two modified PIL interleavers such as PILSS and PILSS2 are given. It is clearly shown that PILLSS and PILSS2 improve the minimum free distance of PIL without disturbing the entire weight spectrum of PIL. Fig 3. Shows the minimum free distance spectrum of PIL, PILSS, and PILSS2 for weight-one information sequences. Fig 4. Shows the minimum free distance spectrum of PIL, PILSS, and PILSS2 for weight-two information sequences. Clearly, PILSS provides the best performance among three interleavers since the last information symbol is sent to the first position of the last row.

In order to verify "error floor" of PIL, PILSS and PILSS2, we investigate HWA with the same approach in [1]. In Table 4, the information patterns considered in calculating HWA asymptote are given. According to the equation (2), HWA is calculated for PIL, PILSS, and PILSS2 for information sequences with Hamming weight of 1-8. In Fig 4 and Fig. 5, PILSS and PILSS2 show better HWA performance than that of PIL. In fact, PIL shows HWA more 10-100 times greater than PILSS and PILSS2.

TABLE 2. INTERLEAVER SIZES FOR MODIFICATION IN PIL INTERLEAVER.

Interleaver sizes	**Interleaver sizes for Modification**	Note
320-2000	600,640,760,840,880,960,1080,1200,1240,1360,1440,1480,1600,1680,1800,1960	
2001-4000	2040,2080,2160,2200,2280,2560,2640,2760,2800,3000,3040,3160,3280,3360,3480,3600,3640,3840,3880,3960,4000	
4001-6000	4240,4480,4560,4600,4680,4800,4840,5040,5160,5280,5400,5440,5560,5640,5680,5880	
6001-8192	6160,6240,6280,6360,6640,6760,6960,7000,7080,7200,7360,7480,7600,7680,7800,7960,8040	

TABLE 3. REPRESENTATIVE MINIMUM FREE DISTANCE OF PIL, PILSS, AND PILSS2 TURBO INTERLEAVER ACCORDING TO INFORMATION HAMMING WEIGHT OF 1 AND 2.

Size K	Weight	PIL	PILSS	PILSS2
600	W_H=1	21,38,48,49,55,57,61,64,65, 74,	37,38,48,49,55,57,61,64,65, 74,	32,37,38,49,55,57,61,64,65, 74,
	W_H=2	37,38,38,42,42,42,42,42,42, 42,	37,38,38,42,42,42,42,42,42, 42,	37,38,38,42,42,42,42,42,42, 42,
1080	W_H=1	21,48,60,61,62,80,84,85,86, 92,	48,53,60,61,62,80,84,85,86, 92,	32,48,50,60,62,80,84,85,86, 92
	W_H=2	42,42,42,42,44,46,46,46,46, 46,	42,42,42,42,44,46,46,46,46, 46,	42,42,42,42,44,46,46,46,46, 46
2040	W_H=1	21,70,75,76,92,104,104,113, 123,132,	70,75,76,80,92,104,104,113, 123,132,	32,70,75,76,81,104,104,113, 123,132
	W_H=2	38,41,42,46,49,50,50,50,50, 50,	41,42,46,49,50,50,50,50,50, 52,	38,41,42,46,49,50,50,50,50, 50,
3040	W_H=1	21,60,83,95,104,111,117,130 ,131,136,	60,83,95,104,109,111,117,13 0,131,136,	32,60,83,95,104,106,111,13 0,131,136,
	W_H=2	38,38,42,46,46,46,48,50,50, 54,	38,38,42,46,46,46,48,50,50, 54,	38,38,42,46,46,46,48,49,50, 54,
4240	W_H=1	21,107,115,142,146,150,152 ,159,168,196,	107,115,141,142,146,150,15 2,159,168,196	32,107,115,141,142,146,150 ,159,168,196,
	W_H=2	33,35,42,45,46,46,50,50,58, 62,	33,35,42,45,46,46,50,50,58, 62,	33,35,42,45,46,46,50,50,58, 62,
5040	W_H=1	21,156,163,164,170,173,185 ,188,190,191,	156,162,163,164,170,173,18 5,188,190,191	32,156,162,163,164,170,185 ,188,190,191,
	W_H=2	46,46,50,51,54,57,58,61,62, 62,	46,46,50,51,54,57,58,61,62, 62,	46,46,50,51,54,58,60,61,62, 62,
6240	W_H=1	21,113,115,122,163,181,194 ,196,211,212,	113,115,122,163,181,194,19 6,200,211,212	32,113,115,122,163,181,194 ,196,201,211,
	W_H=2	42,42,54,62,62,62,77,77,80, 89,	42,42,54,62,62,62,77,77,80, 89,	42,42,54,62,62,62,77,77,80, 89,
6280	W_H=1	21,120,132,133,155,173,174 ,190,193,196,	120,132,133,155,173,174,19 0,193,196,197	32,120,132,133,155,173,174 ,190,193,196,
	W_H=2	39,49,62,65,66,73,73,74,74, 79	39,49,62,65,66,73,73,74,74, 79,	39,49,62,65,66,73,73,74,74, 79,
7200	W_H=1	21,166,172,173,180,181,209 ,214,227,227,	166,172,173,180,181,209,21 4,225,227,227	32,166,172,173,180,181,209 ,214,227,227,
	W_H=2	42,42,46,54,57,58,66,70,70, 70,	42,42,46,54,57,58,66,68,70, 70,	41,42,46,54,57,58,66,70,70, 70,
8040	W_H=1	21,106,106,109,134,182,199 ,215,240,251,	106,106,109,134,182,199,21 5,240,249,251	32,106,106,109,134,182,199 ,215,240,251,
	W_H=2	46,50,53,54,54,58,60,61,65, 66,	46,49,50,53,54,54,58,60,61, 65,	46,50,53,54,54,58,60,61,65, 66,

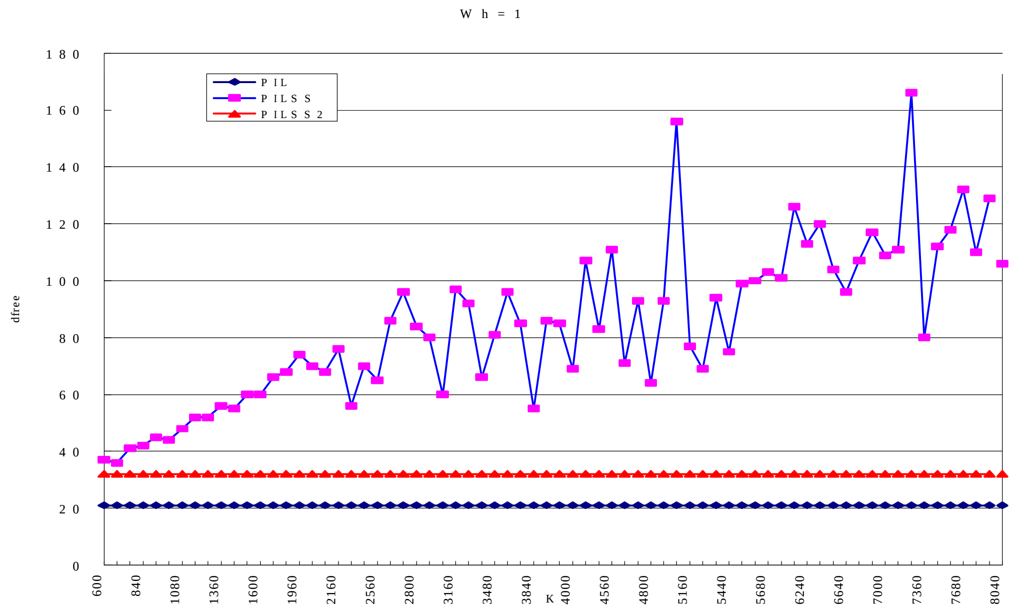

Fig. 3. Minimum free distance of PIL, PILSS, and PILSS2 with information Hamming weight of one.

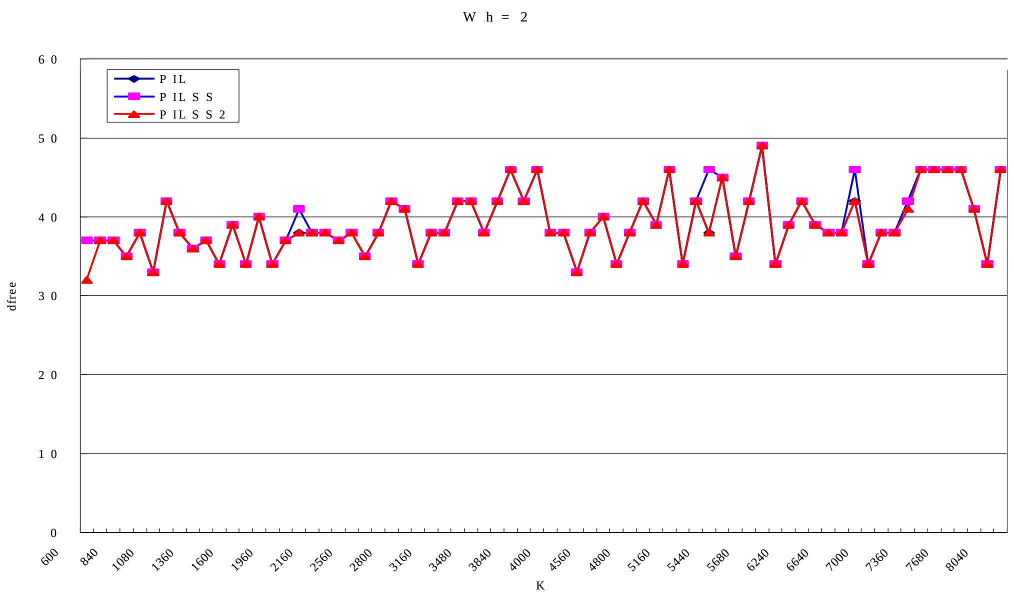

Fig.4. Minimum free distance of PIL, PILSS, and PILSS2 with information Hamming weight of two

TABLE 4. INFORMATION PATTERNS USED FOR CALCULATING HWA.

INFORMATION WEIGHT	INFORMATION WORD PATTERNS
1	D^l , l=0,1,2 ….
2	$1+D^l$, l = 7,14,21,28, …….
3	$1+D+D^{5+l}$, l=0,7,14,21,28, ……
4	$1+D+D^2+D^{4+l}$, l=0,7,14,21,28, ……
5	$1+D+D^2+D^3+D^{8+l}$, l=0,7,14,21,28, …….
6	$1+D+D^2+D^3+D^4+D^{10+l}$, l=0,7,14,21,28, …….
7	$1+D+D^2+D^3+D^4+D^5+D^{6+l}$, l=0,7,14,21,28, ………
8	$1+D+D^2+D^3+D^4+D^5+D^6+D^7$

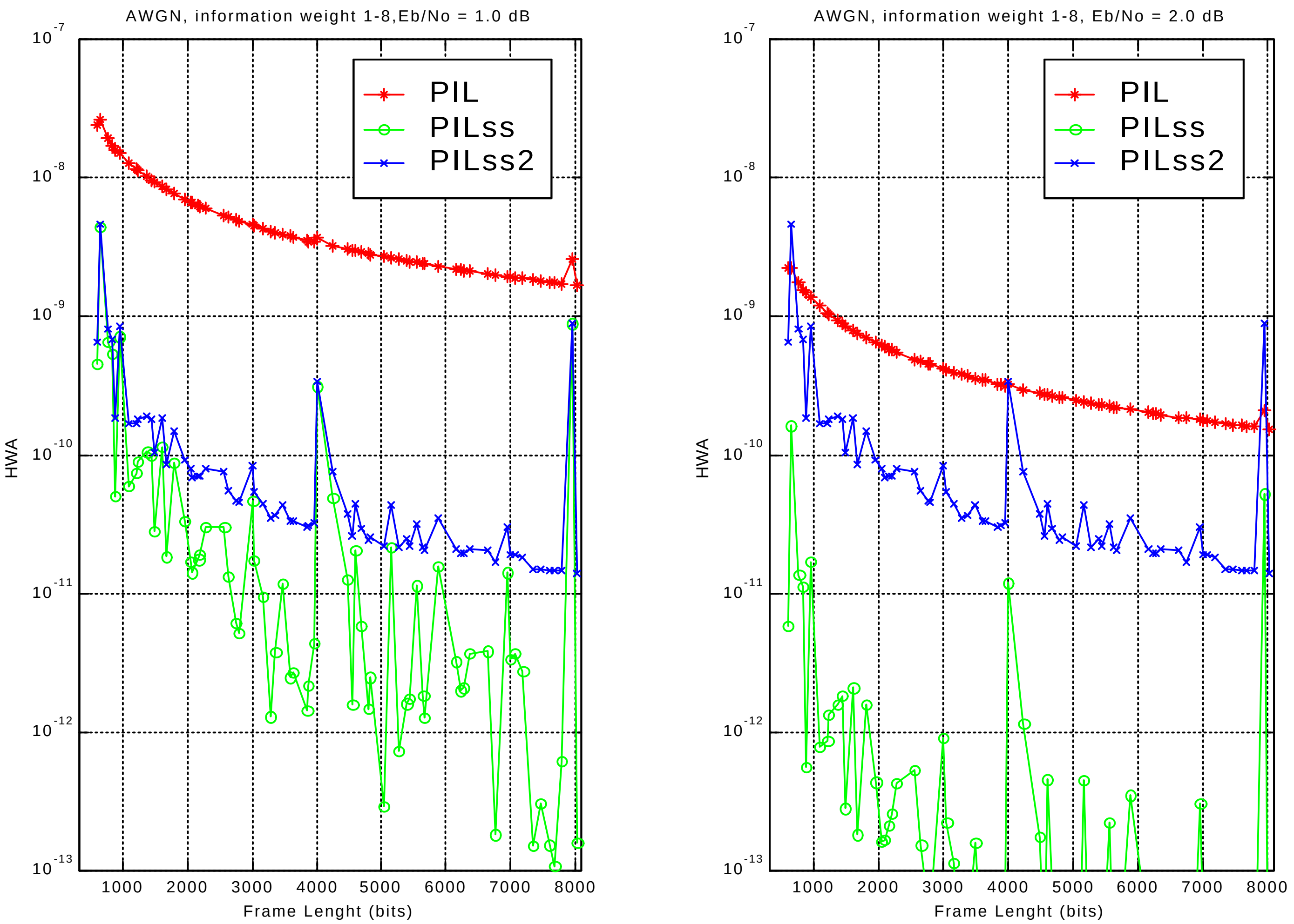

Fig. 4. HWA of PIL, PILSS, and PILSS2 with information Hamming weight of 1-8.

4. Conclusion

In this document, we analyzed the effect of weight-one information sequence for Turbo codes when input information sequence is terminated with the separated zero state termination. Also, we have improved "Hamming weight asymptote (HWA)" by reflecting weight-one information sequence. Weight spectrums of some interleaver candidates showed good consistency with the improved HWA. It has been shown that weight-one information sequences should also be considered as a critical pattern to prevent "error floor" in designing Turbo interleaver with a finite input block size. Based on the results of weight spectrum analysis, We conclude that PIL interleaver may have the performance degradation for some specific interleaver sizes. This kind of degradation should be avoided by improving PIL interleaving algorithm. Also, we proposed two modifications of PIL interleavers, which have shown better performance than the original PIL.

5. References

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