

**Agenda Item:** 4  
**Source:** NTT DoCoMo  
**Title:** A Study on Turbo-interleaver Flexibility  
**Document for:** Discussion

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## 1. Introduction

Channel coding of IMT2000 is required a flexibility to the size of frame which should be changed according to the services. The range of the frame sizes is assumed to be from 320 to 8192 bits. So, in such a wide range of frame sizes, turbo-interleaver is required to provide good permutation performance or low error-floor capability. The purpose of this report is evaluation of turbo interleaver performance in terms of error-floor under the condition of various frame sizes whose range is from 320 to 8192 bits.

## 2. Error-floor estimation using Hamming weight asymptote comparison

We use “hamming weight asymptote” for the evaluation of the error-floor performance instead of BER simulation. Because it is required too much computation time to simulate BER performance of all the frame sizes from 320 to 8129 bits at the moderate and high  $E_b/N_0$  region (error-floor region). Hamming weight asymptote is an estimate for the error-floor of turbo codes based on the union bound. And its estimation error and computational complexity are both small.

### (1) Hamming weight asymptote

It is well known that the lower Hamming weights (including minimum Hamming weight) are the major components for BER calculation in the union bound. One approximation of the union bound using only the minimum Hamming weight is called “Free distance asymptote”, which was introduced in [1] for evaluation of error-floor of turbo codes. In this report, we employ an extension of Free distance asymptote in which the lower (or more) Hamming weights are used for error-floor estimations and we call this “HWA: Hamming Weight Asymptote” in this document. So, it is easy to understand HWA show more accurate estimation of error-floor of turbo codes than Free distance asymptote under AWGN channels. In the following, definition of HWA is described in detail.

In reference [1] and [2], the union bound (an upper bound of BER)  $P_b$  of a  $N$ -bit-length turbo code with the maximum-likelihood (ML) decoding on an additive white Gaussian noise (AWGN) channel is expressed by

$$P_b \leq \sum_{d=d_{free}}^{2(n+N)} \frac{N_d \tilde{w}_d}{N} Q\left(\sqrt{d \frac{2RE_b}{N_0}}\right), \quad (1)$$

where  $d_{free}$  is the free distance of the code,  $\tilde{w}_d$  is the average information weight per code word,  $N_d$  is the number of codewords of weight  $d$ ,  $R$  is coding rate,  $n$  is the number of tail bit and  $Q(x)$  is defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{u^2}{2}\right) du. \quad (2)$$

We use the input distance with weight 2 ( $=\tilde{w}_d$ ) listed in Table 1 which results in the lower Hamming weight of the generated codewords [3]. Thus, the values of the lower weight of 8-state turbo code of rate 1/3 are 14, 18, 22, 26, 30, 34, and 38.

Table 1. Input distance with weight 2 that results in lower Hamming weight. [3]

Weight-2 $1+D^l$	
input length	I
8	7
15	14
22	21
29	28

We calculated the number of the codewords in case that all RSCs suffers form the error events shown in Table 1 and obtained the HWA from Eq. (1).

(2) Hamming weight asymptote and BER performance

HWAs of turbo codes using GF [4] (denoted by GF2), MIL [5], and reference interleaver for frame size of 256 to 4000 bits were calculated and shown Fig. 1. The reference interleaver was the random interleaver offered by Hughes Network Systems. It shows HWA of GF and MIL is lower by one or more decades than that of reference interleaver in many frame sizes. However, there exist some frame size regions, such as frame size s of around 450, 900, 1800, and 2700 bits, where HWA of GF is larger than that of the reference interleaver.

Figure 2 shows the simulated BER and frame error rate (FER) performance of GF, MIL, the reference interleaver, and S-random [6]. HWAs of both GF and MIL are shown in Fig. 2. It shows

- HWA is a accurate estimation for error-floor of turbo codes
- There exist frame size where BER and FER performance of GF interleaver is degraded down to the performance of the random interleaver.

The pruning scheme is considered to be a cause of the degradation of GF interleaver.

### 3. Conclusion

In order to study about Turbo-interleaver flexibility, we evaluated error floor for two Turbo interleavers: GF interleaver and Turbo-MIL, which were already shown that they are applicable for any interleaving bit sizes, using Hamming weight asymptote comparison. From the results of error floor evaluations, it is found that GF interleaver has the performance degradation for specific interleaver bit sizes. These facts may introduce the performance degradations if we want to add a new service with such specific sizes. This kind of degradations should be avoided in IMT2000 systems because one aim of those systems is to realise the multimedia services full of variety.

### References

[1] Lance C. Perez, Jan Seghers and Daniel J. Costello, "A distance spectrum interpretation of turbo codes," IEEE Trans. on Information Theory, vol. 42, No. 6, pp. 1698-1710, Nov. 1996

[2] "An alternative to use of parallel concatenated codes for UMTS: Serially concatenated convolutional codes with interleavers", Lucent Technologies, Tdoc SMG2 UMTS L1 655/98

[3] "Low complexity algebraic interleaver for UTRA turbo codes", Nortel Nteworks, Tdoc SMG2 UMTS L1 051/99

[4] "Description of the GF Interleaver for Turbo codes", Hughes Network Systems, Tdoc SMG2 UMTS L1 765/98

[5] "Description of Multi-stage InterLeaver (MIL) for 8-state Turbo codes", NTT DoCoMo, Tdoc SMG2 UMTS L1 027/99

[6] D. Divsalar and F. Pollara, "Multiple turbo codes for deep-space communications," TDA Progress Report 42-121, pp. 66-77, May 15, 1995.

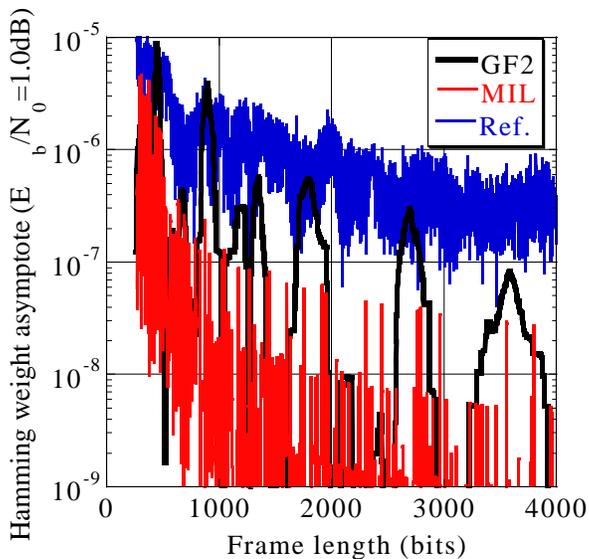
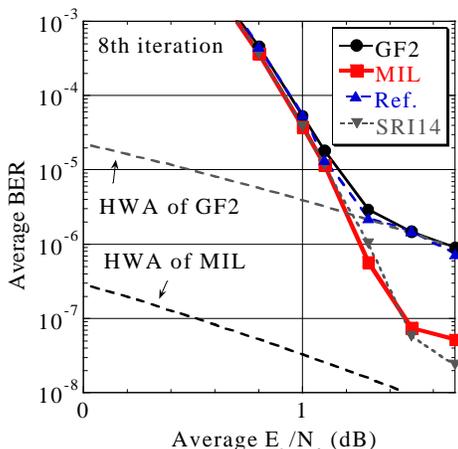
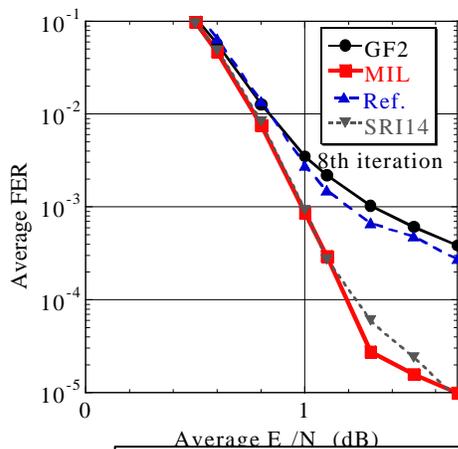


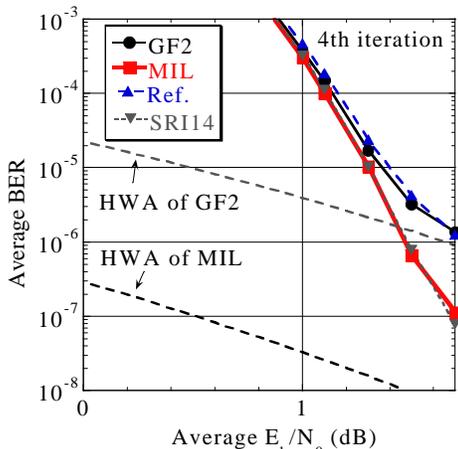
Fig. 1 The Hamming weight asymptote comparison of GF2 and MIL.



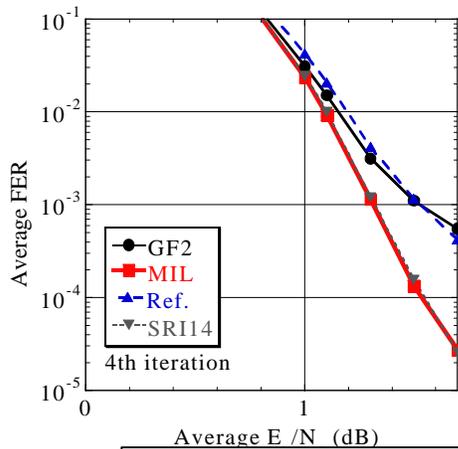
(a) BER of 8-iteration



(b) FER of 8-iteration



(c) BER of 4-iteration



(d) FER of 4-iteration

Fig. 2 The BER/FER performance comparison of GF2 and MIL with 896-bit under AWGN.