

**Reduced complexity Primary and secondary synchronisation codes with good aperiodic correlation properties for the WCDMA system**

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**Summary:**

The 3Gpp S1.13 on FDD [1], spreading and modulation uses the primary (PSC) and secondary synchronization code (SSC) words based upon a hierarchical sequence and length 256 Hadamard sequences respectively. The complexity of stage 1 of acquisition for the above codes is 30 complex adds per correlation output of the input data. Further, as has been brought to notice by Nortel [4] that the aperiodic cross correlation from the SSC's to the PSC for the current sequences is significant. This could impact stage 1 of acquisition, depending upon the number of slots chosen for averaging during stage 1. In this proposal we consider alternative sequences for the PSC and the SSC based upon the Golay sequences and show that they have the following advantages:

- (1) The complexity of stage 1 of acquisition is reduced by half to 15 complex adds per correlation output. Further, a detailed comparison of the current and proposed PSC schemes based upon the number of adds and the number of memory read/write shows that the *proposed scheme should give an estimated 40 % savings in power consumption of stage 1 of acquisition for the current scheme*. Further, the aperiodic auto correlation properties of the proposed Golay PSC are better than the currently used Hierarchical PSC.
- (2) For both *the one slot, eight slot averaging* during stage 1, the SSC aperiodic cross correlation to PSC is much superior for the proposed sequences compared to the currently used PSC, SSC sequences and also the Nortel proposed SSC sequences. *The expected improvement in the performance of stage 1 for proposed sequences over currently used sequences is between 0.9 to 2.6 dB depending upon the comma free code of the base station.*
- (3) The complexity of stage 2 of acquisition is unchanged for the proposed SSC sequences.
- (4) The only required change in the WCDMA standard is a specification for different PSC and SSC codes. The basic structure of the synchronization channel (SCH) is unchanged; thus the overall change in the WCDMA standard is very small.

***Thus, since the proposed PSC and SSC sequences give a 40 % savings in the power consumption for stage 1 of acquisition and much improved aperiodic cross correlation properties of SSC to PSC, they are a better choice over the currently used sequences for PSC and SSC.***

**1.0 Current PSC and SSC's**

A block diagram of the current PSC and SSC's is shown in figure (1).

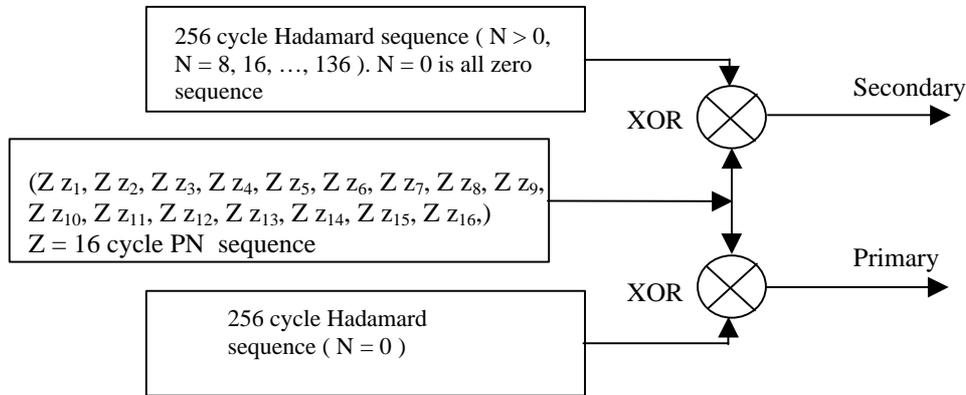


Figure (1): The currently used PSC and SSC's are shown. The sequence  $Z = \{z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_{10}, z_{11}, z_{12}, z_{13}, z_{14}, z_{15}, z_{16}\}$  is a length 16 Lindner sequence given by  $\{1, 1, -1, -1, -1, -1, 1, -1, 1, 1, -1, 1, 1, 1, -1, 1\}$ .

The correlator for despreading the PSC is given in figure(2).

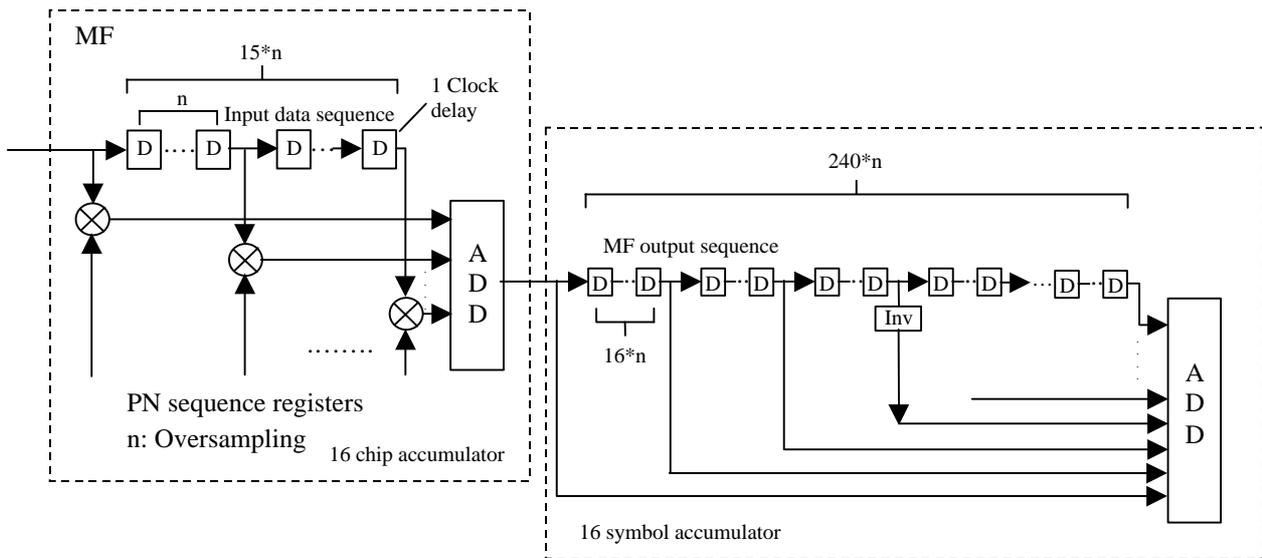


Figure 2: Correlator to despread the Hierarchical PSC. We can see that the total number of complex adds needed for each correlation output is  $(16+16) = 30$  adds per output sample.

**2.0 The Golay sequences**

A description of the Golay sequence and the efficient Golay correlator (EGC) is given in [2, 3]. For completeness, we repeat the description given in [2].

The Golay sequences can be constructed for any length  $L=2^N$ , where  $N$  is any positive integer, and also for lengths 10 and 26, or for any combination of those three lengths. The Golay sequences have a low aperiodic auto-correlation sidelobes, and there is a large number of them for a given code length.

If the sequences are of length  $L=2^N$ , there is a general method for the construction of polyphase complementary pairs of sequences, where the Golay complementary sequences are just a special, binary case. That general construction is defined by the following recursive relation [3].

$$\begin{aligned} a_0(k) &= \delta(k) \\ b_0(k) &= \delta(k) \\ a_n(k) &= a_{n-1}(k) + W_n \cdot b_{n-1}(k-D_n) \\ b_n(k) &= a_{n-1}(k) - W_n \cdot b_{n-1}(k-D_n), \end{aligned} \quad (1)$$

$$\begin{aligned} k &= 0, 1, 2, \dots, 2^N-1, \\ n &= 1, 2, \dots, N, \\ D_n &= 2^{P_n}, \end{aligned}$$

where

- $a_n(k)$  and  $b_n(k)$  are two complementary sequences of length  $2^N$ ,
- $\delta(k)$  is the Kronecker delta function,
- $k$  is an integer representing the time scale,
- $n$  is the iteration number,
- $D_n$  is a delay,
- $P_n, n = 1, 2, \dots, N$ , is any permutation of numbers  $\{0, 1, 2, \dots, N-1\}$ ,
- $W_n$  is an arbitrary complex number of unit magnitude.

If  $W_n$  has values  $+1$  and  $-1$ , the binary (Golay) complementary sequences are obtained [3]. An efficient matched filter directly corresponding to the complementary sequences  $a_N(k)$  and  $b_N(k)$  defined by (1) is given in Figure 3. This filter performs the correlation of input signal  $r(k)$  *simultaneously* with the two complementary sequences  $a_N(k)$  and  $b_N(k)$ . The two matched filter outputs produce the two corresponding aperiodic cross-correlation functions  $R_{ra}(\tau)$  and  $R_{rb}(\tau)$ . Such a digital filter will be called the Efficient Golay Correlator (EGC), although it is actually the filter matched also to any polyphase complementary pair defined by (1). *For the binary Golay sequences of interest to us,  $W_n = +/- 1$  implying that no actual multiplies are needed for the binary Golay sequences.*

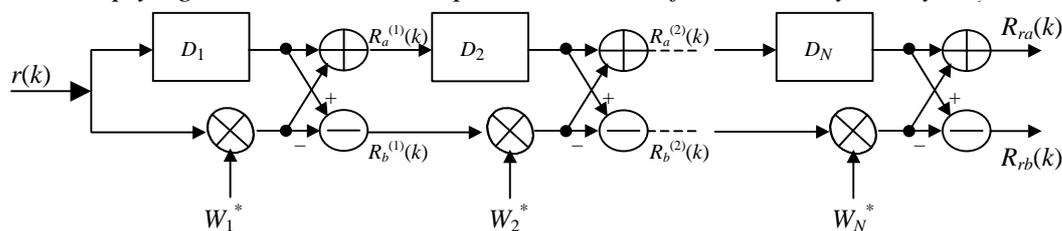


Figure 3: Efficient Golay Correlator (EGC). The total number of complex adds to despread a length 256 binary Golay sequence is  $2\log(256) = 16$  per output sample. This is approximately half the number of adds needed for the hierarchical sequence (figure 2).

The boxes in Figure 3 represent the corresponding delay lines with  $D_n$  memory elements. The number of *additions* in the EGC is  $2 \cdot \log_2(L)$ , while in the hierarchical sequence (figure 1) is  $2\sqrt{L}-2$ . The number of memory elements required for the EGC is  $L-1$  ( $=D_1+D_2+\dots+D_N$ ), the same as for the hierarchical sequence (figure 2).



The new block diagram for the PSC and SSC's is given in following figure 4:

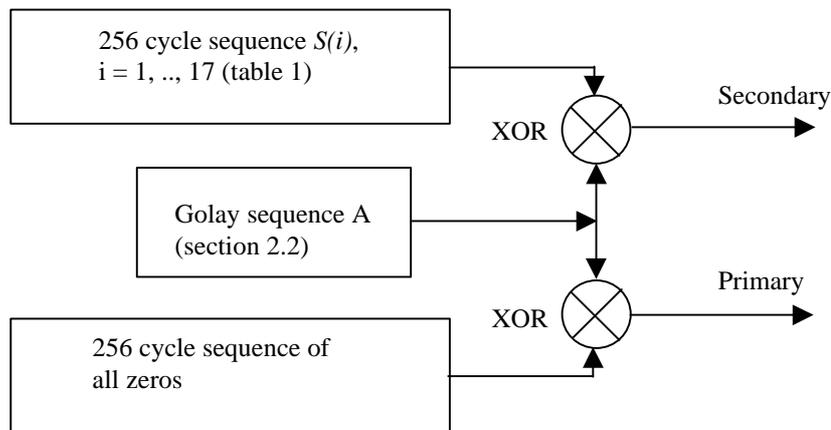


Figure 4: Block diagram of the proposed PSC and SSC's.

### 3.0 Comparison of the current and the proposed sequences

A number of different parameters are used to compare the current (figure 1) and the proposed (figure 4) synchronization sequences namely;

- A. Aperiodic autocorrelation of the PSC's (relevant for stage 1 of acquisition)
- B. A periodic cross correlations of the SSC's to PSC (relevant for stage 1 of acquisition)
- C. Complexity of the receiver in terms of the total delay elements and the number of real adds per output correlation sample for the primary and secondary stages of synchronization.

We now compare the current and proposed sequences according to the criteria A, B and C given above.

#### 3.1 Criteria A: Aperiodic autocorrelation of the PSC

Figure 5 plots the aperiodic autocorrelation for the hierarchical and the proposed Golay sequences (section 2.2):

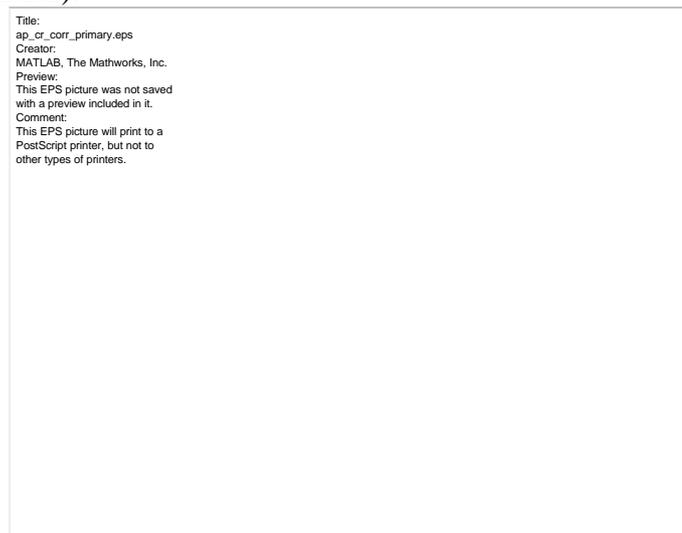


Figure 5: Plot of aperiodic autocorrelation of the current and the proposed PSC  
 The statistical properties of the aperiodic auto correlation are summarized in table 2:

	Hierarchical sequence (figure 1)	Golay sequence (section 2.2)
Maximum a periodic autocorrelation (off peak)	34	27
Root mean square of the aperiodic autocorrelation (off peak)	8.0	6.5

Table 2: Comparison of the statistical properties of the aperiodic autocorrelation (off peak) of the current and proposed sequence. We can see that the proposed Golay sequence has better autocorrelation properties compared to the Hierarchical sequence.

As can be seen from figure 5 and table 1, the off peak aperiodic auto correlation of the proposed Golay sequence is better than the current Hierarchical sequence.

### 3.2 Criteria B: Aperiodic cross correlation of SSC's to PSC

Figure 6 plots the the aperiodic cross correlation for all the Hadamard SSC's to the Hierarchical PSC used currently



Figure 6: Aperiodic cross correlation of the currently used Hadamard SSC's to the currently used Hierarchical PSC. There are a total of 256 points per SSC, hence the total number of points on the x-axis is  $17 \times 256 = 4352$ .

Of interest are also the SSC sequences proposed by Nortel based upon Hierarchical sequences [4]. Figures 7, 8 plot the aperiodic cross correlation of Nortel proposed SSC's (Hierarchical) to Hierarchical PSC and to Golay PSC respectively.

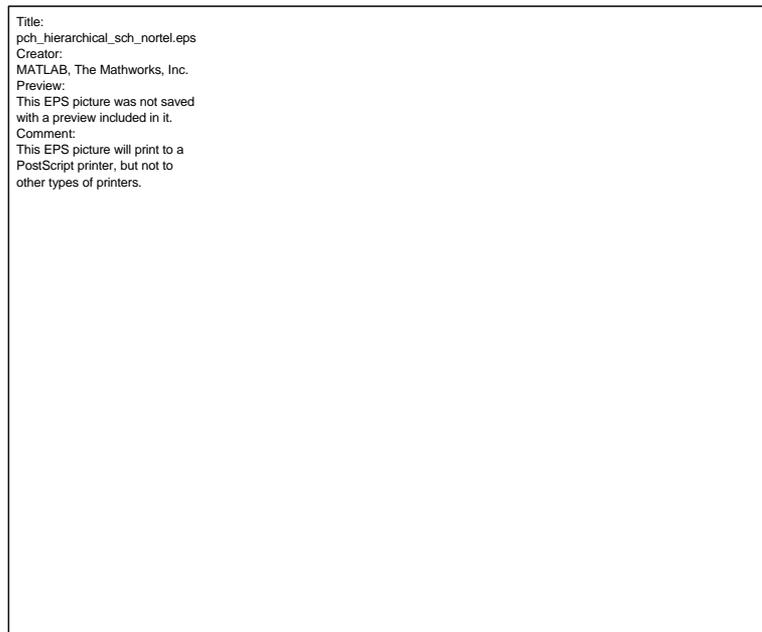


Figure 7: *Aperioidic cross correlation of the Nortel proposed SSC's to the Hierarchical PSC. There are a total of 256 points per SSC, hence the total number of points on the x-axis is  $512 * 256 = 131072$ .*

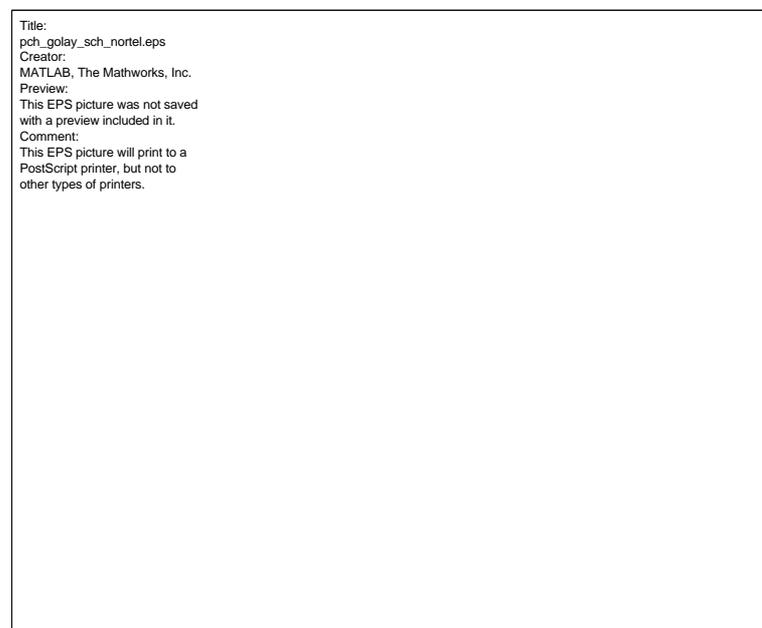


Figure 8: *Aperioidic cross correlation of the Nortel proposed SSC's to the Proposed Golay 256 PSC. There are a total of 256 points per SSC, hence the total number of points on the x-axis is  $512 * 256 = 131072$ .*

Figure 9 plots the aperioidic cross correlation for all the proposed Golay SSC's (table 1) to the proposed Golay PSC.

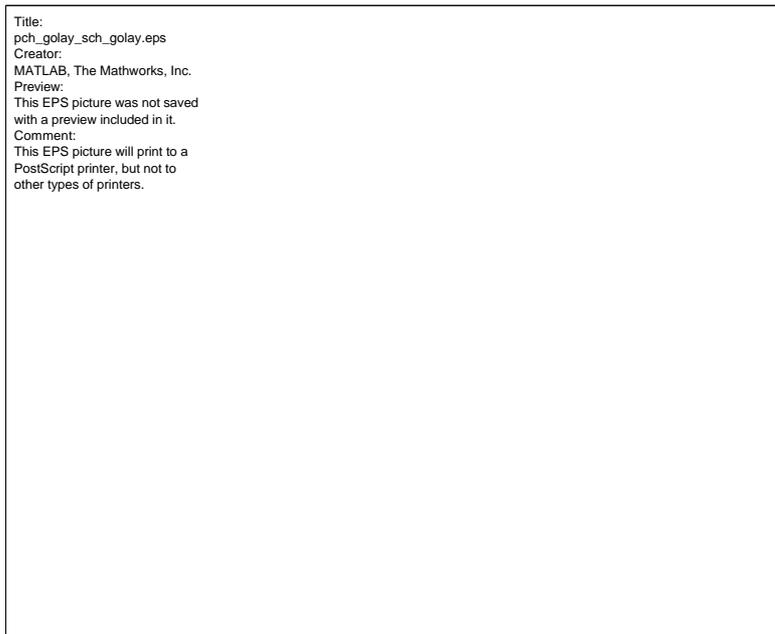


Figure 9: Aperiodic cross correlation of the proposed SSC's to the proposed PSC. There are a total of 256 points per SSC, hence the total number of points on the x-axis is  $17 \times 256 = 4352$ . The statistical properties of the aperiodic SSC to PSC cross correlation is summarized in table 3:

	PSC: Hierarchical SSC: Hadamard (Currently used)	PSC: Hierarchical SSC: Hierarchical (Nortel)	PSC: Golay SSC: Hierarchical (Nortel)	PSC: Golay SSC: Golay (Proposed)
Maximum a periodic cross correlation from the SSC's to PSC	176	111	97	49
Root mean square of the aperiodic cross correlation of SSC's to PSC	10.7	11.5	11.4	11.4

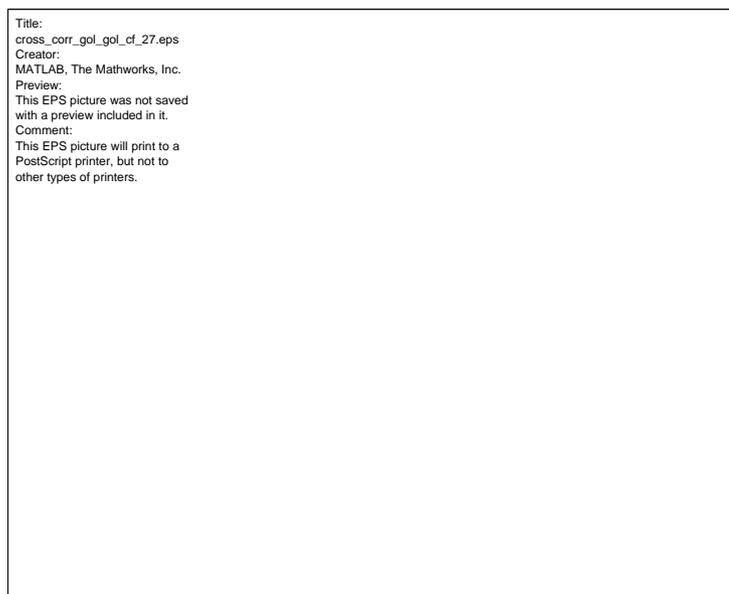
Table 3: Comparison of the statistical properties of the aperiodic cross correlation of the SSC's to the PSC for the current and the proposed systems is shown.

We can see from table (3), that the peak cross correlation for the current sequences and the Nortel proposed sequences is much higher than the proposed sequences. On the other hand, the root mean square of the cross correlation is about the same for both the cases. During acquisition, it is the peak cross correlation which will give rise to false alarms. Hence we can see that the proposed sequences should perform better than the current sequences, or the ones based upon the Nortel sequences *even for one slot averaging*. In practice, for stage 1 of acquisition, a number of slots are non-coherently averaged implying that the cross correlation actually gets averaged out because of the different comma free codes transmitted on the SSC. Since in the Nortel proposed scheme, the same sequence is repeated in every slot, the proposed sequences will further outperform the Nortel proposed sequences. Thus henceforth, we do not compare the proposed sequences against the Nortel proposed sequences.

The aperiodic cross correlation of PSC and SSC adds during stage 1 acquisition. Thus we now plot the square root of the off peak aperiodic cross correlation of (PSC+SSC) with PSC, non-coherently averaged (by squaring) over the 8 *time slots* for the 27<sup>th</sup> comma free code in figure (10) and (11). The peak output correlation, not shown in the figure is 256.



*Figure 10: Square root of the off-peak aperiodic cross correlation of the (Hadamard SSC+Hierarchical PSC ) to Hierarchical PSC during stage 1 of acquisition for the non-coherent averaging (by squaring) over 8 time slots and for all 16 time slot shifts is shown for the 27<sup>th</sup> comma free code. There are a total of 255 correlation outputs per shift, hence the total number of points on the x-axis is  $255 \cdot 16 = 4080$ . The peak output (not shown) is 256.*



*Figure 11: Square root of the off-peak aperiodic cross correlation of the (Golay SSC+Golay PSC ) to Golay PSC during stage 1 of acquisition for the non-coherent averaging (by squaring) over 8 time slots and for all 16 time slot shifts is shown for the 27<sup>th</sup> comma free code. There are a*

*total of 255 correlation outputs per shift, hence the total number of points on the x-axis is 255\*16 = 4080. The peak output (not shown) is 256.*

The statistical properties of the averaged aperiodic cross correlation from (SSC+PSC) to PSC by averaging over 8 time slots *for the full comma free code of length 16* is summarized in table 4:

Maximum off-peak aperiodic cross correlation from the (SSC+PSC) to PSC	Currently used sequences ( $\alpha$ )	Proposed sequences ( $\beta$ )	<i>Expected gain for proposed sequences (dB)</i> $20*\log_{10}((256-\beta)/(256-\alpha))$
Comma free code 1	68	47	<b>0.9</b>
Comma free code 2	76	44	<b>1.4</b>
Comma free code 3	85	46	<b>1.8</b>
Comma free code 4	87	43	<b>2.0</b>
Comma free code 5	93	45	<b>2.3</b>
Comma free code 6	89	42	<b>2.2</b>
Comma free code 7	72	43	<b>1.3</b>
Comma free code 8	87	43	<b>2.1</b>
Comma free code 9	82	43	<b>1.8</b>
Comma free code 10	86	43	<b>2.0</b>
Comma free code 11	88	40	<b>2.2</b>
Comma free code 12	73	44	<b>1.3</b>
Comma free code 13	68	46	<b>1.0</b>
Comma free code 14	88	44	<b>2.0</b>
Comma free code 15	88	47	<b>1.9</b>
Comma free code 16	92	39	<b>2.5</b>
Comma free code 17	90	44	<b>2.1</b>
Comma free code 18	87	45	<b>1.9</b>
Comma free code 19	79	43	<b>1.6</b>
Comma free code 20	84	42	<b>1.9</b>
Comma free code 21	76	46	<b>1.3</b>
Comma free code 22	92	43	<b>2.2</b>
Comma free code 23	70	43	<b>1.2</b>
Comma free code 24	87	45	<b>1.9</b>
Comma free code 25	63	43	<b>0.9</b>
Comma free code 26	92	43	<b>2.3</b>
Comma free code 27 <i>(shown in figures 10, 11)</i>	96	38	<b>2.6</b>
Comma free code 28	86	46	<b>1.8</b>
Comma free code 29	82	44	<b>1.7</b>
Comma free code 30	87	45	<b>1.9</b>
Comma free code 31	87	42	<b>2.0</b>
Comma free code 32	89	45	<b>2.0</b>

*Table 4: Comparison of the statistical properties of the off-peak aperiodic cross correlation of the (SSC+PSC) to the PSC for the current and the proposed sequences is shown with the main peak normalized to 256. An 8 slot averaging is assumed for each comma free code of length 16. The probability of false alarm during stage 1 will be dominated by the maximum aperiodic cross correlation. Hence the criterion given in the third column, the ratio of the difference of the main peak and the maximum aperiodic cross correlation for the current and proposed sequences, should be an indicator of amount of gain to be*

expected from the proposed sequences. We can see that gain for the proposed sequences varies between 0.9 to 2.6 dB.

We can see from tables 3, 4, that the proposed sequences have much superior maximum aperiodic cross correlation as compared to the current sequences, even after significant averaging over 8 time slots for the stage 1 of acquisition. The Root mean square of the aperiodic cross correlation after 8 slot averaging for the 32 comma free codes varies between 11.9 to 13.9 for the current sequences. While for the proposed sequences it varies between 12.9 to 13.4, implying that it is almost the same for the current and the proposed sequences. During stage 1 of acquisition, the probability of false alarm will be dominated by the maximum aperiodic cross correlation. Hence, the performance of the proposed sequences should be superior compared to the currently used PSC and SSC's. **As shown in table 4, the expected performance improvement for the proposed sequences over the current sequences varies between 0.9 to 2.6 dB depending upon the comma free code of the base station.**

### 3.3 Criteria D: Complexity of the receiver

#### 3.3.1 Complexity of stage 1

The number of adds required for stage 1 for the current PSC is 30 (figure 2) per output sample. The number of adds for the proposed sequence is 15 per output sample (1 less than  $2 \log_2 256$  because we are interested in correlation with sequence A in figure 3 ) which is half of that required for the current PSC. However, in order to compare the power consumption for the two schemes, we should also take into account memory accesses required by the two schemes. Figure 12 shows a rearranged block diagram of figure 2 for despreading the current PSC for oversampling  $n = 1$  that is one sample per chip. For a simplicity of initial comparison of the two schemes assume that a length 256 memory is used to implement all the delay elements, as shown in figure 12. For the current scheme, we now need 30 adds, 30 memory reads and 2 memory writes per correlation output for the current PSC scheme.

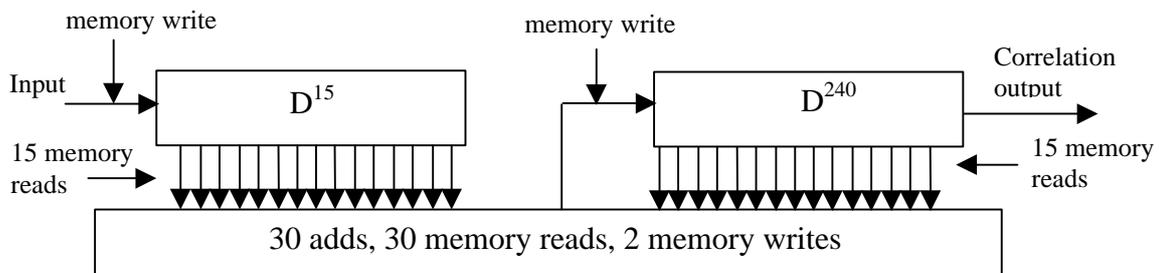


Figure 12: Implementation of the correlator for the current PSC implemented using memory elements. We can see that we need 30 adds, 30 memory reads, 2 memory writes per correlation output.

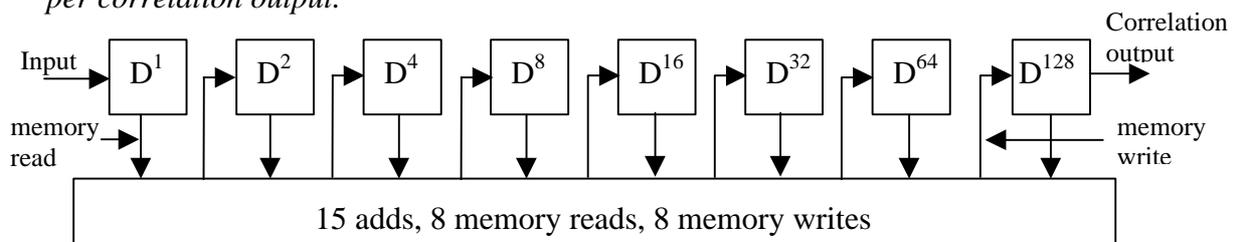


Figure 13: *Implementation of the correlator for the proposed PSC implemented using memory elements. We can see that we need 15 adds, 8 memory reads, 8 memory writes per correlation output.*

Figure 13 shows a rearranged block diagram of figure 3 for despreading the proposed PSC for oversampling  $n = 1$  that is one sample per chip. Again assuming that a length 256 memory is used to implement all the delay elements, as shown in figure 13, we need 15 adds, 8 memory reads and 8 memory writes per correlation output for the proposed PSC scheme. Table 5 compares the estimated power consumption for the current and the proposed schemes using the figures 12, 13. We can see that the estimated power savings for the proposed scheme is 40 % of the total power consumption for the current scheme. As shown in the table, if the power consumption for stage 1 of acquisition is 20, 10 or 5 mW, then the power consumption for the proposed scheme will be 12, 6 and 3 mW respectively.

	Hierarchical sequence (figure 1)	Golay sequence (figure 4)
Number of delay elements	255	255
Number of adds per output sample	30	15
Number of memory reads per output sample	30	8
Number of memory writes per output sample	2	8
Power consumption for one add	$x$	$x$
Power consumption for one memory read	$x$	$x$
Power consumption for one memory write	$2x$	$2x$
Power consumption of adds per output sample	$30x$	$15x$
Power consumption of memory reads per output sample	$30x$	$8x$
Power consumption of memory writes per output sample	$4x$	$16x$
<b>Total power consumption per output sample</b>	<b><math>64x</math></b>	<b><math>39x</math></b>
<b>Reduction in power consumption for proposed scheme over current scheme</b>		<b><math>(1 - 64/39) = 40\%</math></b>
Power consumption for stage 1 of acquisition	if, 20 mW	then, 12 mW
Power consumption for stage 1 of acquisition	if, 10 mW	then, 6 mW
Power consumption for stage 1 of acquisition	if, 5 mW	then, 3 mW

Table 5: *Estimated power consumption savings for the proposed scheme over the current scheme is shown. We assume the power consumption for an add is approximately equal to the power consumption of a memory read and half that of a memory write. We can see that the proposed scheme is estimated to give 40 % power savings over the current scheme during stage 1 of acquisition respectively.*

### 3.3.2 Complexity of stage 2

We first compute the complexity of stage 2 for the current SSC. The complexity for generating the correlation outputs per time slot corresponding to the 17 comma free codes is given in table 6:

32 correlations for 8 chips	$7*32 = 224$ complex adds
17 correlations using length 32 Walsh Hadamard transform	$32*\log_2(32) - 15 = 145$ complex adds
Total complexity	$(224 + 145) = 369$ complex adds

Table (6): *Complexity of stage 2 for the current Hadamard sequences.*

Let us now try to calculate the total complexity of stage 2 for the SSC's derived from Golay sequences (table 1).

- (1) In the first step the mobile needs to generate the 32 correlations for the sets of 8 chips with respect to both the sequence A and B. Each EGC needs  $2 \cdot \log_2(8) = 6$  complex adds. Thus the total operations for the 32 correlations is  $6 \cdot 32 = 192$  complex adds. Let the correlation outputs be denoted by  $a_1, a_2, \dots, a_{32}$  and  $b_1, b_2, \dots, b_{32}$ .
- (2) The second step is the combining of these correlation outputs to generate the correlation with respect to the sequences given in table (1). We first consider the generation of correlation of correlation outputs of  $a_1, a_2, \dots, a_{16}$  with respect to the sequences  $X_1, X_2, \dots, X_{16}$  in table 1. Figure 13 illustrates this computation. Even though it is not exactly like an FFT, the total number of operations can be seen to be exactly the same as a length 16 FFT =  $16 \cdot \log_2 16 = 64$ . The correlations with respect to  $X_{17}$  can be generated using 15 brute force additions. Thus the total computations for correlation outputs of  $a_1, a_2, \dots, a_{16}, b_1, b_2, \dots, b_{16}$  with respect to the sequences  $X_1, X_2, \dots, X_{16}, X_{17}$  are  $(64 + 15 = 79)$  complex adds. In addition we require equal number of operations for correlations of  $a_{17}, a_{18}, \dots, a_{32}, b_{17}, b_{18}, \dots, b_{32}$  with respect to the sequences  $X_1, X_2, \dots, X_{16}, X_{17}$ . Thus the total number of operations for this step so far are  $2 \cdot 79 = 158$ . These correlations now have to be combined to generate 17 correlations with respect to  $S(0), S(1), \dots, S(16)$  in table 1 requiring 17 extra additions. Thus the number of operations for this step so far is;  $158 + 17 = 175$  complex adds.
- (3) Adding complexity in steps (1) and (2), we now get the total complexity for the stage 2 of proposed SSC's to be  $(192 + 175 = 367)$  complex adds.

Comparing the above complexity to that of the current sequences (table 6), we can see that the complexities are essentially the same.

***Thus, for the same complexity of stage2 and about half the complexity of stage 1, we can get improved stage 1 acquisition performance for the proposed sequences.***

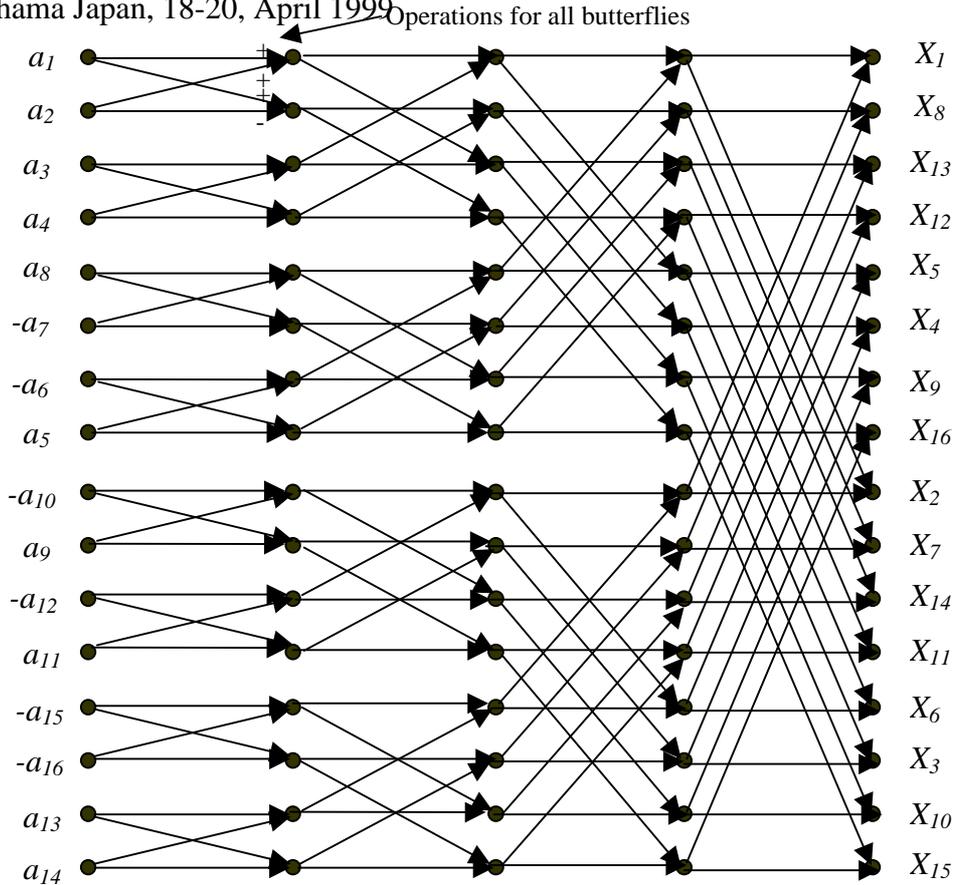


Figure 13: FFT like computation for the generating the correlations of  $a_1, a_2, \dots, a_{16}$  with respect to the sequences  $X_1, X_2, \dots, X_{16}$ .

#### 4.0 Conclusions:

We have shown that the proposed PSC and SSC have superior aperiodic auto correlation and cross correlation properties compared to the current sequences (and the Nortel proposed sequences), which could affect the stage 1 of acquisition. The expected gain in the performance of stage 1 for proposed sequences over the currently used sequences even after *eight slot averaging* varies between **0.9 to 2.6 dB**, depending upon the comma free code of the base station. Further, the improved correlation properties are obtained at an estimated **40 % savings in power for stage 1 of acquisition** and no increase in complexity for stage 2 of acquisition.

***Thus, since the proposed PSC and SSC sequences give a 40 % savings in the power consumption for stage 1 of acquisition and much improved aperiodic cross correlation properties of SSC to PSC, they are a better choice over the currently used sequences for PSC and SSC.***

#### References:

1. 3Gpp S1.13, FDD spreading and modulations specification.
2. TSGR1#3(99)205, Nynashamn Sweden, Ericsson, New RACH preambles with low auto-correlation sidelobes and reduced detector complexity.
3. S.Z.Budisin, "Efficient pulse compressor for Golay complementary sequences", Electronics Letters, Vol.27, No.3, pp.219-220, Jan. 1991.
4. TSGR 1 # 2 (99)090, Yokohama Japan, Nortel Networks, Synchronization channel with cyclic Hierarchical sequences.