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Source: Motorola Mobility
Title:
Spatial Correlation Accuracy with WINNER II Large Scale Parameter Generation
Document for: Discussion

1. Introduction

The 3D channel model large scale parameters (LSPs), delay spread, shadow fading, *K* factor, and angular spreads of angles of arrival and departure in azimuth and zenith, are, before any non-linear transformation are applied, a set of jointly Gaussian random variables with specified cross-correlations and exponential spatial autocorrelation functions of the form $R(d)=\mathrm{e}^{-\frac{d}{d_{cor}}}$ where *d* is the distance between two points in the horizontal plane and d_{cor} is the LSP's correlation distance. The 3D channel model TR does not specify a method or algorithm for generating the LSPs although 3GPP IMT Advanced simulations used the method specified in WINNER II [1] [2].

The spatial autocorrelation function of LSPs generated according to the WINNER II method is shown below through both analysis and simulation to *not* decay exponentially and may in fact become small at a distance different than its correlation distance.

2. Generation of Correlated LSPs in WINNER II

The WINNER II method for correlating LSPs is described in Figure 1. For each of *M* LSPs, the procedure generates a spatially correlated Gaussian random field, or map, at a set of grid points by filtering a set of i.i.d Gaussian random variables defined on the same set of grid points. The filter used to generate the *m*th map is of the form in Eq. 3.12 of Figure 1 and is therefore a function of the *m*th LSP's correlation distance Δ_m. The *M* sets of i.i.d random variables are uncorrelated and therefore the resulting *M* spatially correlated maps are uncorrelated (meaning every point in one map is uncorrelated with any point in another map). Presumably through interpolation, the LSPs at a set of *K* UE positions are generated for each map and stacked into the set of vectors $\boldsymbol{\xi}_M(x_k, y_k)$, $k=1,2,\cdots,K$. The vector of LSPs, $\tilde{\mathbf{s}}(x_k, y_k)$ is then pre-multiplied by a matrix square root of the desired LSP correlation matrix to obtain *K* length-*M* vectors representing the *M* LSPs of all *K* users.

Note that each component of $\boldsymbol{\xi}_M(x_k, y_k)$ was generated with a filter whose coefficients were determined by the correlation distance corresponding to that components LSP, i.e. the *m*th component of $\boldsymbol{\xi}_M(x_k, y_k)$ seems to correspond to LSP *m*. However this correspondence is broken when the *M* components are mixed by the matrix multiplication in Eq. 3.14 of Figure 1. After this matrix multiplication, the *m*th LSP's spatial autocorrelation function is a linear combination of all spatial autocorrelation function of all *M* LSPs:

$$\mathbf{E}\left[\tilde{s}_m(x_k, y_k)\tilde{s}_m(x_{k'}, y_{k'})\right] = \sum_{m'=1}^{M}\left|\sqrt{C_{MM}(0)}(m,m')\right|^2 \mathbf{exp}\left(-d(x_k, y_k; x_{k'}, y_{k'})/\Delta_m\right) \quad (1)$$

The cross-correlation and autocorrelation of the LSPs are seen to be coupled with the WINNER II method, i.e. the choice of the cross-correlations affect the autocorrelation.

The Appendix derives the result in Eq. 1 for the case of $M = 2$.

Sec. 3.3.1 of WINNER II D1.1.2 Part 1

Let us assume we have M LSPs per link and K correlated links, i.e. K MSs linked to the same BS site at locations (x_k, y_k), where $k = 1,\ldots,K$. Auto-correlation is generated to the LSPs the following way. At first we generate a uniform grid of locations based on co-ordinates of the K MSs. Size of the grid is $(\max(x_k) - \min(x_k) + 2D) \times (\max(y_k) - \min(y_k) + 2D)$. To each grid node we assign M Gaussian iid ~N(0,1) random numbers, one for each LSP. Then the grid of random numbers is filtered with a two dimensional FIR filter to generate exponential auto-correlation. Impulse response of the filter for the mth LSP is

$$h_m(d) = \exp\left(-\frac{d}{\Delta_m}\right), \quad (3.12)$$

where d is distance and Δ_m is the correlation distance both in meters (see Table 4-5). Each of the M random numbers in nodes of the grid, representing M LSPs, is filtered with a specific filter, because the correlation distances may be different in Table 4-5. After filtering the correlated random numbers $\boldsymbol{\xi}_M(x_k, y_k)$ at K grid nodes (K MS locations) are saved and the redundant grid nodes are discarded.

Cross-correlation is generated independently to the LSPs of K links by linear transformation

$$\tilde{\mathbf{s}}(x_k, y_k) = \sqrt{\mathbf{C}_{MxM}(0)}\boldsymbol{\xi}_M(x_k, y_k), \quad (3.13)$$

where elements of correlation matrix

$$\mathbf{C}_{MxM}(0) = \begin{bmatrix} C_{\tilde{s}_1\tilde{s}_1}(0) & \cdots & C_{\tilde{s}_1\tilde{s}_M}(0) \\ \vdots & \ddots & \vdots \\ C_{\tilde{s}_M\tilde{s}_1}(0) & \cdots & C_{\tilde{s}_M\tilde{s}_M}(0) \end{bmatrix} \quad (3.14)$$

are defined in Table 4-5.

Figure 1: Method of Correlating LSPs in WINNER [1].

3. Simulation Results

Simulation results for the case of $M = 2$ LSPs with correlation $\rho = 0.75$ are shown in Figure 2. The correlation distances of LSPs 1 and 2 are 10 and 30 respectively. One choice for the square root of the correlation matrix is the Cholesky decomposition:

$$\sqrt{C_{MM}(0)} = \begin{bmatrix} 1 & 0 \\ \frac{3}{4} & \frac{\sqrt{7}}{4} \end{bmatrix}.$$

Since this matrix is lower triangular, the first LSP will, from Eq. (1), be exponential with the correct correlation distance. This is shown by the red line in Figure 2. The second LSP on the other hand will have autocorrelation

$$\mathbf{E}\left[\tilde{s}_2(x_k, y_k)\tilde{s}_2(x_{k'}, y_{k'})\right] = \frac{9}{16}\mathbf{exp}\left(-d(x_k, y_k; x_{k'}, y_{k'})/10\right) + \frac{7}{16}\mathbf{exp}\left(-d(x_k, y_k; x_{k'}, y_{k'})/30\right)$$

as seen in the magenta line. The desired autocorrelation function of LSP 2 ('LSP 2 Desired' shown in solid blue line) decays much slower, reaching e^{-1} (.3679) at about twice the value of the realized autocorrelation distance ('LSP 2 Simulation' shown circle blue circles and blue line).

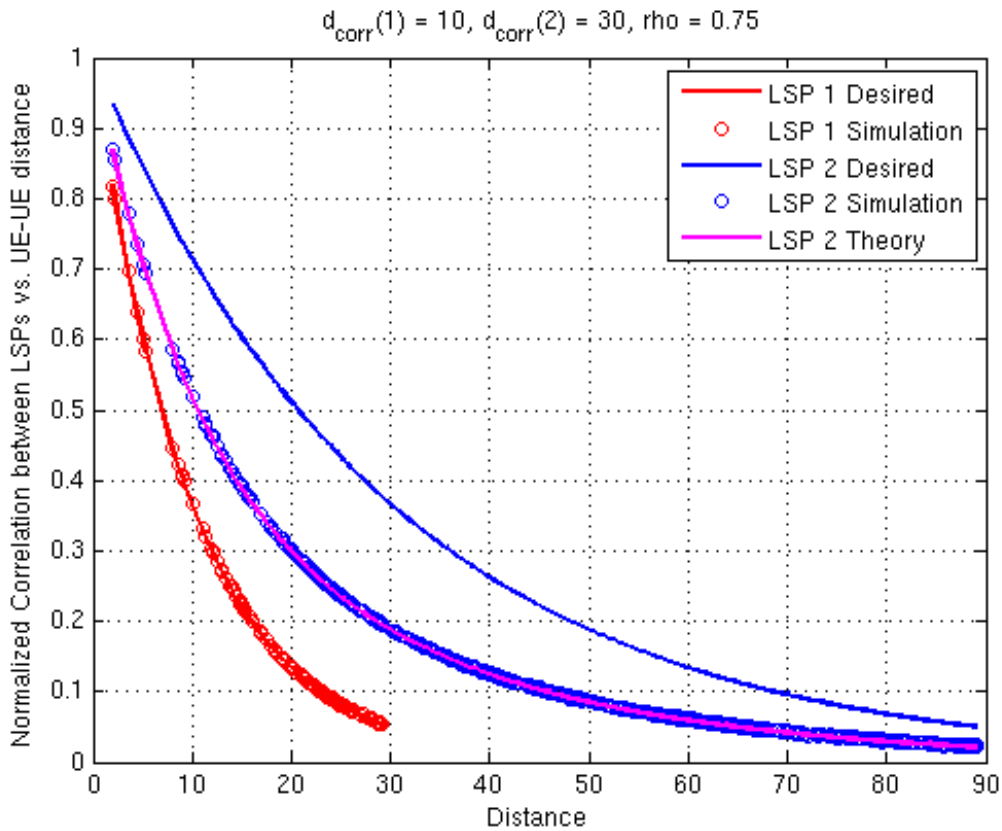

Figure 2: Normalized autocorrelations for 2 LSPs with inter-parameter correlation of $\rho = 0.75$ and correlation distances of 10 and 30. LSP 1's autocorrelation decays exponentially at the correct rate as expected. LSP 2 on the other hand should decay exponentially with a correlation distance of 30, i.e. reaches e^{-1} (.3679) at 30 (as shown solid blue line for the 'LSP 2 desired'). However due to the coupling between inter-parameter correlations and spatial autocorrelation, the decay rate of the realized LSP 2 ('LSP 2 Simulation' shown circle blue line) is not exponential and reaches e^{-1} at only around 15.

4. Recommendation

Ideally, an algorithm for generating LSPs with both the desired inter-parameter correlations and spatial autocorrelation functions would have been available at the beginning of the study item. Unless a reference to such an algorithm can be found and companies are willing to do another round of simulations, it is probably best at this point to simply document the algorithm that was used by the majority of companies in

generating correlated LSPs. If the WINNER II method is such an algorithm, then the following points should be kept in mind:

1) The square root of the correlation matrix, $\sqrt{C_{MM}(0)}$, is not unique. For example, simply permuting the order of LSPs in the two LSP case will give different autocorrelation functions for both LSPs with and without permutation.
2) Even without considering permutations, the matrix square root of a positive definite matrix is not unique. For example, in addition to the Cholesky decomposition, the eigenvector decomposition could also be used and would result in completely different autocorrelation functions than with the Cholesky decomposition.

Given these observations, documentation of the WINNER II method of LSP generation in the TR should include the following in Step 4 of Sec. 7.3 as a means to at least provide some level of repeatability to the LSP generation process.

- A reference to Sec. 3.3.1 of the WINNER II channel document (or some other reference) should be provided.
- The method used to determine the correlation matrix square root (e.g., Cholesky decomposition, as used in WINNER II), should be noted.
- The ordering of the LSPs within the vector of LSPs (e.g., the LSP vector which appears on the left hand side of Eq. 3.3.1 of [1] the WINNER II document) should be specified.

5. Conclusions

The WINNER II method of introducing cross-correlation between LSPs and providing spatial auto-correlation was shown to generate LSPs with spatial autocorrelation functions which are not exponential and do not decay in distance at the designed rate. This is due to the coupling between inter-parameter and spatial correlation inherent in the algorithm.

Barring a desire by companies to rerun Phase 2 calibration and baseline simulation results based on a different LSP generation algorithm, it is recommended that any LSP generation algorithm used by a majority of companies should be documented in the specification. If the WINNER II method, which was specified for IMT Advanced simulations, was used by a majority of the companies, then this method should be documented according to the following:

- A reference to the Sec. 3.3.1 of the WINNER II channel document should be provided.
- The use of the Cholesky decomposition, as used in WINNER II, should be noted.
- The ordering of the LSPs within the vector of LSPs which appears on the left hand side of Eq. 3.3.1 of [1] should be specified.

6. Appendix

According to (3.12) of Section 3.3.1 of the WINNER II D1.1.2 Part 1, M uncorrelated grids of uncorrelated Gaussian i.i.d ~N(0,1) random numbers are filtered by two dimensional FIR filters to obtain $\xi_1 \; x,y$ and $\xi_2 \; x,y$ where here we assume the number of LSPs is $M=2$ and we use a continuous instead of discrete grid. We therefore have

$$E \; \xi_m \; x,y \; \xi_m \; x',y' \; = \exp \; -d \,/\, \Delta_m \; , \qquad m = 1,2$$

and

$$E\ \ \xi_1\ \ x,y\ \ \zeta_2\ \ x',y'\ \ = 0; \quad \text{for all} \quad x,y,x',y'\ \ .$$

with

$$d = \sqrt{x - x'\ ^2 + \ y - y'\ ^2}$$

Next, cross-correlation between LSPs is introduced by linear transformation of $\xi_1\ \ x,y$ and $\zeta_2\ \ x,y$:

$$\begin{bmatrix} \tilde{s}_1\ \ x,y \\ \tilde{s}_2\ \ x,y \end{bmatrix} = \sqrt{\mathbf{C}} \begin{bmatrix} \xi_1\ \ x,y \\ \xi_2\ \ x,y \end{bmatrix}$$

where $\sqrt{\mathbf{C}}$ is a square root of the matrix $\mathbf{C}$, i.e. $\sqrt{\mathbf{C}}\sqrt{\mathbf{C}}^T = \mathbf{C}$:

$$\sqrt{\mathbf{C}} = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix}.$$

The autocorrelation function of $\tilde{s}_2\ \ x,y$ is then

$$\begin{aligned} E\ \ \tilde{s}_2\ \ x,y\ \ \tilde{s}_2\ \ x',y'\ \ &= E\ \ \ \rho\xi_1\ \ x,y\ \ + \sqrt{1-\rho^2}\xi_2\ \ x,y\ \ \left(\ \rho\xi_1\ \ x',y'\ \ + \sqrt{1-\rho^2}\xi_2\ \ x',y'\ \ \right) \\ &= \rho^2 \exp\ -d\ /\ \Delta_1\ \ +\ 1-\rho^2\ \ \exp\ -d\ /\ \Delta_2 \\ &\neq \exp\ -d\ /\ \Delta_2 \end{aligned}$$

which is a weighted sum of the desired autocorrelation functions of the two LSP and **not** the desired autocorrelation function $\exp\ -d\ /\ \Delta_2$.

7. References

[1] WINNER II D1.1.2 V1.2, WINNER II Channel Models. February 2, 2008.

[2] 3GPP TR 36.814, Further Advancements for E-UTRA Physical Layer Aspects, v9.0.