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1 Introduction

In this contribution we consider precoding codebook design for the 4TX downlink and detail a codebook structure that is suitable for both the uniform linear array (ULA) and the cross-pole antenna configurations. The codebook structure is derived using fundamental properties of the spatial correlation matrices under the ULA and cross-pole antenna configurations. Each precoding codeword is derived as the product of two matrices, as in the 8 TX case. The technical details behind the design are deferred to the appendices.

2 Codebook Construction in product form

We discuss a codebook construction in which each codeword is derived as a matrix product. Letting $\mathbf{w}_n = [1 \ \exp(j2\pi n/16)]^T$ for $n = 0, \cdots, 15$ we define the inner (wideband) codebook as

$$\mathcal{C}^{(1)} = \Bigg\{ \begin{bmatrix} \mathbf{A}(q) \odot \mathbf{W}^{(1)}(k) & \mathbf{0} \\ \mathbf{0} & \mathbf{B}(q) \odot \mathbf{W}^{(1)}(k) \end{bmatrix} : \\ \mathbf{W}^{(1)}(k) = [\mathbf{w}_{2k \bmod 16}, \mathbf{w}_{2k+1 \bmod 16}, \mathbf{w}_{2k+2 \bmod 16}, \mathbf{w}_{2k+3 \bmod 16}], \ k = 0, \cdots, 7; \ q = 1, \cdots, Q \Bigg\}, \quad (1)$$

where $\odot$ denotes Hadamard product and

$$\begin{aligned} \mathbf{A}(q) &= \begin{bmatrix} 1 & 0 \\ 0 & \exp(j2\pi \, d_q) \end{bmatrix} \begin{bmatrix} a_q & a_q & b_q & b_q \\ b_q & b_q & a_q & a_q \end{bmatrix} \\ \mathbf{B}(q) &= \exp(j2\pi\gamma_q) \begin{bmatrix} 1 & 0 \\ 0 & \exp(j2\pi \, d_q) \end{bmatrix} \begin{bmatrix} b_q & b_q & a_q & a_q \\ a_q & a_q & b_q & b_q \end{bmatrix}, \quad q = 1, \cdots, Q, \end{aligned} \quad (2)$$

where $\gamma_q, a_q, b_q, d_q \in [0,1]$ & $a_q^2 + b_q^2 = 1/2 \; \forall \; q$. The rank-1 outer codebook is defined as

$$\begin{aligned} \mathcal{C}_1^{(2)} &= \left\{ \begin{bmatrix} \mathbf{y} \\ \mathbf{y} \end{bmatrix}, \begin{bmatrix} \mathbf{y} \\ -\mathbf{y} \end{bmatrix}, \begin{bmatrix} \mathbf{y} \\ j\mathbf{y} \end{bmatrix}, \begin{bmatrix} \mathbf{y} \\ -j\mathbf{y} \end{bmatrix} \right\} & (3) \\ \mathbf{y} &= \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}, & (4) \end{aligned}$$

where $\mathbf{e}_i$ denotes the 4×1 column selection vector. To obtain the final rank-1 codebook, we choose a 4-bit inner (wideband) codebook using $\gamma_q = d_q = 0, \; \forall \; q$ and $[a_q, b_q] \in \{[0.4352, 0.5573], [0.5000, 0.5000]\}$ so that $Q = 2$. We remark the setting the parameter $\rho = 1/2$ in the exponential correlation model discussed in Appendix C yields $[a_q, b_q] = [0.4352, 0.5573]$, whereas $\rho = 1$ yields $[a_q, b_q] = [0.5, 0.5]$.

For rank-2, the outer rank-2 codebook is defined as

$$\begin{aligned} \mathcal{C}_2^{(2)} &= \left\{ \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 \\ \mathbf{y}_1 & -\mathbf{y}_2 \end{bmatrix}, \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 \\ j\mathbf{y}_1 & -j\mathbf{y}_2 \end{bmatrix} \right\}, \\ (\mathbf{y}_1, \mathbf{y}_2) &= \{(\mathbf{e}_1, \mathbf{e}_1), (\mathbf{e}_2, \mathbf{e}_2), (\mathbf{e}_3, \mathbf{e}_3), (\mathbf{e}_4, \mathbf{e}_4), (\mathbf{e}_1, \mathbf{e}_3), (\mathbf{e}_1, \mathbf{e}_4), (\mathbf{e}_2, \mathbf{e}_3), (\mathbf{e}_2, \mathbf{e}_4)\} \quad (5) \end{aligned}$$

To obtain the final rank-2 codebook, we choose a 5-bit inner (wideband) codebook using

$$[a, b] \in \{[0.4352, 0.5573], [0.5000, 0.5000]\}; d = \gamma/2 \in \{0, 1/4\},$$

so that $Q = 4$.

The rank-3 and rank-4 codebooks can be fixed to the legacy (Householder) rank-3 and rank-4 codebooks.

We note that not all codeword matrices in the aforementioned codebook satisfy the constant magnitude property. However, the codebook has the desirable property that for each rank and for each choice of the inner (wideband) precoder, the expected value of each row norm square (i.e., sum of the magnitude squares of elements in that row) of the final selected precoder matrix is identical, where the expectation is computed over the set of outer (subband) precoders assuming that they are chosen equi-probably.

3 Simulation Results

The main parameters are summarized in Table 1 and the evaluation results are given in Tables 2 through 5. It is seen that the gains obtained via codebook enhancement in the simulated full buffer scenarios are modest. The usage in our wideband codebook is plotted in Fig. 1, where we note that indices (0-to-8) in the rank-1 case and indices (0-to-16) in the rank-2 case correspond to the non-constant modulus codewords. It

Parameter	**Assumption**
Deployment scenario and Channel model	Urban Macro (UMa), scenario A in TR36.871
Duplex method and bandwidth	FDD: 10MHz for downlink
Cell layout	Hex grid 19 sites, 3 cells/site
Transmission power at BS	46 dBm
Number of users per sector	10
Network synchronization	Synchronized
Antenna configuration (eNB)	4 TX cross-polarized/ULA ant., 0.5-λ/4-λ spacing
Antenna configuration (user)	2 RX cross-polarized/ULA ant.
Downlink transmission scheme	Dynamic SU/MU-MIMO scheduling: MU-MIMO pairing: Max 2 users/RB;
Codebook	Rel. 8 codebook/Enhanced Codebooks
Downlink scheduler	PF in time and frequency
Scheduling granularity:	5 RBs
Feedback assumptions	5ms periodicity and 5ms delay; feedback mode 3-2 feedback without errors.
Sub-band granularity:	5 RBs
Downlink HARQ scheme	Chase Combining
Downlink receiver type	LMMSE-IRC
Interference estimation error	Wishart distribution model as in 3gpp.TR36.829
Feedback channel error	NA
Control channel and reference signal overhead	3 OFDM symbols for control; Used TBS tables in TS 36.213

Table 1: Simulation Parameters for Homogenous Networks

is seen that such codewords are indeed selected with non-negligible probabilities which suggests that design of non-constant modulus codebooks should be investigated.

4 Conclusions

In this contribution we detailed a codebook structure and presented an embodiment which conforms to the matrix product form. This structure is motivated by fundamental properties of the spatial correlation matrix and makes codebook optimization feasible. However, the gains obtained over the legacy codebook by all the evaluated enhanced codebooks are rather modest. *An enhanced codebook should be considered only if these gains are deemed sufficient.* In that case the design of non-constant modulus codebooks should also be considered.

4-Tx Codebook	cell average	5% cell-edge
Baseline (LTE Rel-8)	2.8339	0.0861
NEC [2]	2.9907 (5.53%)	0.0873 (1.39%)
Source 1 [3]	2.9324 (3.48%)	0.0842 (-2.21%)
Source 2 [4]	2.9486 (4.05%)	0.0871 (1.16%)
Source 3 [5]	2.9395 (3.73%)	0.0869 (0.93%)
Source 4 [6]	2.9769 (5.05%)	0.0878 (1.97%)

Table 2: Spectral efficiency of various 4Tx enhanced codebooks. Dynamical SU/MU scheduling is employed with near orthogonal paired ZF transmit precoding for MU-MIMO. Relative percentage gains are over the baseline scheme using Rel-8 4Tx codebook. Cross-polarized antennas with $\frac{\lambda}{2}$ spacing, 100% outdoor UEs.

4-Tx Codebook	cell average	5% cell-edge
Baseline (LTE Rel-8)	2.7835	0.0773
NEC [2]	2.9279 (5.19%)	0.0789 (2.07%)
Source 1 [3]	2.8709 (3.14%)	0.0771 (-0.26%)
Source 2 [4]	2.8944 (3.98%)	0.0793 (2.59%)
Source 3 [5]	2.885 (3.65%)	0.0747 (-3.36%)
Source 4 [6]	2.9166 (4.78%)	0.0786 (1.68%)

Table 3: Spectral efficiency of various 4Tx enhanced codebooks. Dynamical SU/MU scheduling is employed with near orthogonal paired ZF transmit precoding for MU-MIMO. Relative percentage gains are over the baseline scheme using Rel-8 4Tx codebook. Cross-polarized antennas with $\frac{\lambda}{2}$ spacing, 20% outdoor UEs.

References

[1] Ericsson,ST-Ericsson, "Design and Evaluation of 4 TX Precoder Codebooks for CSI Feedback," *3GPP TSG RAN WG1 R1-104847* 62, Madrid, August 2010.

[2] NEC Group, "On the 4TX Codebook Enhancement," *3GPP TSG RAN WG1 R1-130845*, RAN1#72, St Julian's, Malta, Jan. 2013.

[3] Ericsson,ST-Ericsson, "Four antenna port codebook," *3GPP TSG RAN WG1 R1-130855*, RAN1#72, St Julian's, Malta, Jan. 2013.

[4] NTT DOCOMO, "Enhanced Codebook Design for Downlink 4-Tx MIMO," *3GPP TSG RAN WG1 R1-130846*, RAN1#72, St Julian's, Malta, Jan. 2013.

[5] Samsung, "A proposal for four-antenna-port codebook," *3GPP TSG RAN WG1 R1-130850*, RAN1#72, St Julian's, Malta, Jan. 2013.

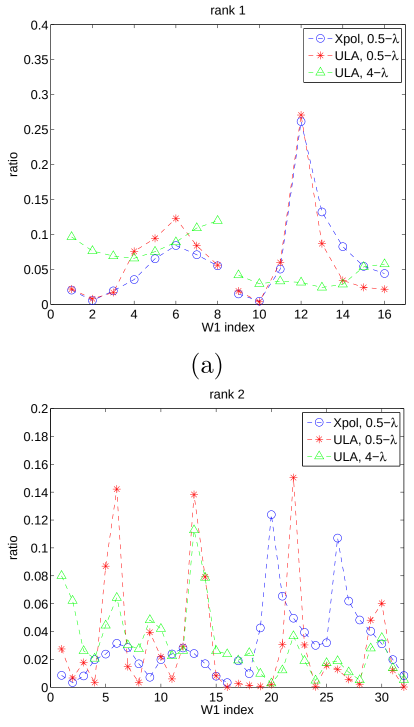
[6] Renesas Mobile Europe Ltd., "Double codebook for 4Tx antennas," *3GPP TSG RAN WG1 R1-130854*, RAN1#72, St Julian's, Malta, Jan. 2013.

4-Tx Codebook	cell average	5% cell-edge
Baseline (LTE Rel-8)	2.6426	0.0840
NEC [2]	2.7836 (5.34%)	0.0942 (12.14%)
Source 1 [3]	2.6818 (1.48%)	0.0849 (1.07%)
Source 2 [4]	2.7761 (5.05%)	0.0970 (15.48%)
Source 3 [5]	2.6955 (2.00%)	0.0891 (6.07%)
Source 4 [6]	2.7992 (5.93%)	0.0901 (7.26%)

Table 4: Spectral efficiency of various 4Tx enhanced codebooks. Dynamical SU/MU scheduling is employed with near orthogonal paired ZF transmit precoding for MU-MIMO. Relative percentage gains are over the baseline scheme using Rel-8 4Tx codebook. ULA antennas with 4-λ antenna spacing, 100% outdoor UEs.

4-Tx Codebook	cell average	5% cell-edge
Baseline (LTE Rel-8)	2.5375	0.087
NEC [2]	2.6524 (4.53%)	0.0898 (3.22%)
Source 1 [3]	2.5539 (0.65%)	0.0823 (-5.40%)
Source 2 [4]	2.6427 (4.15%)	0.0885 (1.72%)
Source 3 [5]	2.5646 (1.07%)	0.0812 (-6.67%)
Source 4 [6]	2.6571 (4.71%)	0.0836 (-3.91%)

Table 5: Spectral efficiency of various 4Tx enhanced codebooks. Dynamical SU/MU scheduling is employed with near orthogonal paired ZF transmit precoding for MU-MIMO. Relative percentage gains are over the baseline scheme using Rel-8 4Tx codebook. ULA antennas with 4-λ antenna spacing, 20% outdoor UEs.



(a)

(b)

Figure 1: Usage of wideband codebook: (a) Rank-1 (b) Rank-2.

A Uniform Linear Array

In the following, unless otherwise mentioned we will assume the co-polarized antennas to be closely spaced. We have the following observations for the uniform linear array (ULA) transmit antenna configuration. Consider a system with N co-polarized transmit antennas and let $\mathbf{C}$ denote the transmit spatial correlation matrix. Let us define $\mathbf{J}$ to be the matrix which has zeros everywhere except on the cross diagonal elements, i.e., $\mathbf{J} = [J_{m,n}]$ where

$$J_{m,n} = \begin{cases} 1, & n = N - m + 1 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

A vector is said to be Hermitian if

$$\bar{\mathbf{x}} = \mathbf{J}\mathbf{x}, \tag{7}$$

where $\bar{\mathbf{x}}$ denotes the conjugate of $\mathbf{x}$. We offer the following set of properties:

Lemma 1 *The matrix* $\mathbf{C}$ *is a* Hermitian Toeplitz *matrix, i.e.,* $\mathbf{C}$ *satisfies*

$$\bar{\mathbf{C}} = \mathbf{J}\mathbf{C}\mathbf{J}, \tag{8}$$

where $\bar{\mathbf{C}}$ *denotes the conjugate of* $\mathbf{C}$*.*

Lemma 2 *The eigenspace of any Hermitian Toeplitz matrix can be completely described by Hermitian vectors. In other words, given a Hermitian Toeplitz matrix* $\mathbf{A}$ *and with* $\mathbf{x}$ *being its eigenvector such that* $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$*, then* $\exists\ \mathbf{y}$ *such that* $\bar{\mathbf{y}} = \mathbf{J}\mathbf{y}$ *with*

$$\mathbf{A}\mathbf{y} = \lambda\mathbf{y} \tag{9}$$

Lemma 3 *Suppose* λ *is an eigenvalue of a Hermitian Toeplitz matrix* $\mathbf{A}$ *with algebraic multiplicity one. Then if* $\mathbf{x}$ *is an eigenvector such that* $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$*, we must have*

$$\mathbf{J}\mathbf{x} = \exp(j\delta)\bar{\mathbf{x}}. \tag{10}$$

for some $\delta \in [0, 2\pi)$ *and where* $j = \sqrt{-1}$*.*

A simplified model for the correlation matrix is the exponential correlation model, which is discussed further in the Appendix, and is given by

$$\mathbf{C} = [C_{m,n}]_{m,n=1}^{N},\ C_{m,n} = \rho^{|m-n|}\exp(j\theta(m-n)),\ m,n \in \{1,\cdots,N\}, \tag{11}$$

where $\rho \in [0,1]$ & $\theta \in [0,2\pi)$.

A.1 4 TX ULA

In this section, we consider the case of $N = 4$ co-polarized transmit antennas. First, note that without loss of generality, we can impose the following structure on each eigenvector $\mathbf{x}$ of the spatial correlation matrix $\mathbf{C}$,

$$\mathbf{x} = [a_1, a_2\exp(j\theta_2), a_3\exp(j\theta_3), a_4\exp(j\theta_4)]^T, \tag{12}$$

where $a_1, a_2, a_3, a_4 \in I\!R_+$. Recalling that the matrix $\mathbf{C}$ must be a Hermitian Toeplitz matrix and invoking Lemmas 2 and 3, we can deduce that we must have

$$\begin{aligned} a_1 = a_4,\ a_2 = a_3 \\ \theta_2 + \theta_3 = \theta_4. \end{aligned} \tag{13}$$

Then, consider any two eigen-vectors of the form in (13) given by

$$\mathbf{x} = [a, b\exp(j\theta_2), b\exp(j\theta_3), a\exp(j(\theta_2+\theta_3))]^T \tag{14}$$

$$\mathbf{y} = [c, d\exp(j\gamma_2), d\exp(j\gamma_3), c\exp(j(\gamma_2+\gamma_3))]^T, \tag{15}$$

where $a, b, c, d \in I\!R_+$. Then, a sufficient condition to enforce orthogonality among these two eigenvectors is to ensure that

$$\begin{aligned} \theta_2 + \theta_3 &= \pm\pi + \gamma_2 + \gamma_3 \\ \theta_2 - \theta_3 &= \pm\pi + \gamma_2 - \gamma_3 \end{aligned}$$

which can be simplified to

$$\begin{aligned}\theta_2 &= t\pi + \gamma_2, t \in \{0, \pm 1\} \\ \theta_3 &= \pm(1 - |t|)\pi + \gamma_3. \end{aligned} \tag{16}$$

We remark that (16) is not necessary but works for all possible values of the scalars $a, b, c, d \in \mathbb{R}_+$.

B Polarized Setup

Suppose the transmitter has $2N$ cross-polarized antennas comprising of a pair of N co-polarized antennas each. Then the correlation matrix of each one of these two co-polarized sets is denoted by $\mathbf{C}$ which is Hermitian and Toeplitz. The overall $2N \times 2N$ correlation matrix $\tilde{\mathbf{C}}$ can be written as

$$\tilde{\mathbf{C}} = \begin{bmatrix} 1 & \alpha \\ \bar{\alpha} & 1 \end{bmatrix} \otimes \mathbf{C} \tag{17}$$

where $\otimes$ denotes the kronecker product and $\alpha \in \mathbb{C} : |\alpha| \in [0, 1]$. It can be shown that any eigenvector $\tilde{\mathbf{y}}$ of $\tilde{\mathbf{C}}$ has the form

$$\tilde{\mathbf{y}} = \mathbf{y} \otimes \mathbf{x} \tag{18}$$

where $\mathbf{y} \in \mathbb{C}^{2\times 1}$ is an eigenvector of the matrix $\begin{bmatrix} 1 & \alpha \\ \bar{\alpha} & 1 \end{bmatrix}$ and $\mathbf{x}$ is an eigenvector of $\mathbf{C}$. Furthermore, the two eigenvectors of the matrix $\begin{bmatrix} 1 & \alpha \\ \bar{\alpha} & 1 \end{bmatrix}$ are $\frac{\exp(j\beta)}{\sqrt{2}}[1 \quad \frac{\alpha}{|\alpha|}]^T$ and $\frac{\exp(j\beta)}{\sqrt{2}}[1 \quad -\frac{\alpha}{|\alpha|}]^T$, where $(.)^T$ denotes the transpose operation and the phase term $\exp(j\beta)$ can be ignored without loss of optimality. The two eigenvalues are $1 \pm |\alpha|$. Note that the matrix $\begin{bmatrix} 1 & \alpha \\ \bar{\alpha} & 1 \end{bmatrix}$ also models the correlation matrix of the 2 transmit ULA.

C 4 TX ULA with the exponential correlation model

Next, we consider the case where we further specialize the correlation matrix to be

$$\mathbf{C} = \begin{bmatrix} 1 & a & a^2 & a^3 \\ b & 1 & a & a^2 \\ b^2 & b & 1 & a \\ b^3 & b^2 & b & 1 \end{bmatrix}, \tag{19}$$

where $a \in \mathbb{C}$ such that $|a| \leq 1$ and $b = \bar{a}$. Note that the matrix $\mathbf{C}$ is Hermitian Toeplitz and is also completely characterized by one complex scalar. Thus its eigenvectors can be expected to have more structure in addition to that possessed by an eigenvector of a general Hermitian Toeplitz matrix. We will also exploit this additional structure in the following.

We first consider the case $|a| < 1$. In this case, the eigen-vectors of any matrix of the form in (19) have the following properties. Consider any matrix $\mathbf{C}$ of the form in (19) and let

$$\mathbf{C} = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^\dagger \tag{20}$$

denote its eigen-decomposition where $(.)^\dagger$ denotes the conjugate transpose operation and $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ with $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ denoting the four real-valued eigenvalues. Then,

$$\mathbf{E} = \mathbf{D}_p(\mathbf{H} \odot \mathbf{S}) \tag{21}$$

where $\odot$ denotes the Hadamard product and $\mathbf{D}_p$ is a diagonal matrix of the form

$$\mathbf{D}_p = \mathrm{diag}\{1, \exp(j\gamma), \exp(2j\gamma), \exp(3j\gamma)\} \tag{22}$$

for some $\gamma \in [0, 2\pi)$. The matrix $\mathbf{S}$ has the following structure

$$\mathbf{S} = \begin{bmatrix} p & r & q & s \\ q & s & p & r \\ q & s & p & r \\ p & r & q & s \end{bmatrix} \tag{23}$$

for some real positive scalars p, q, r, s such that $q = \sqrt{1/2 - p^2}$ and $r = \sqrt{1/2 - s^2}$. The matrix $\mathbf{H}$ is a 4×4

real-valued Hadamard matrix, i.e., columns of $\mathbf{H}$ are mutually orthogonal and all its elements belong to the set $\{\pm 1\}$. An important example $\mathbf{H}$ is the following:

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}$$

Using $\mathbf{H}$ given above and recalling that $\rho = |a|$, we can derive formulas that yield the scalars p, q, r, s as follows. First we have that $\exp(j\gamma) = \frac{\bar{a}}{|a|}$. Substituting this in (21) yields after some manipulations that

$$q = \frac{\rho(1+\rho)}{\sqrt{2[\rho^2(1+\rho)^2 + (\vartheta - \rho)^2]}}, \tag{24}$$

where $\vartheta = \frac{\rho+\rho^3+\sqrt{\rho^2(1+\rho^2)^2+4\rho^2(1+2\rho)}}{2}$ and $p = \sqrt{1/2 - q^2}$. Note that in the special case when $\rho = 0$ the correlation matrix $\mathbf{C}$ reduces to the identity matrix so that we can chose p, q, r, s arbitrarily (subject to the respective norm constraints). Further, for $\rho < 1$, we can determine that $r = \sqrt{\frac{\zeta+\rho}{2(2\zeta+\rho+\rho^3)}}$ with $s = \sqrt{1/2 - r^2}$ where $\zeta = \frac{-\rho-\rho^3+\sqrt{(\rho-\rho^3)^2+4\rho^2(1-\rho)^2}}{2}$.

On the other hand when $\rho = 1$ we note that the matrix $\mathbf{C}$ is a rank-1 matrix that is given by

$$\mathbf{C} = \begin{bmatrix} 1 \\ b \\ b^2 \\ b^3 \end{bmatrix} \begin{bmatrix} 1 & a & a^2 & a^3 \end{bmatrix}. \tag{25}$$

It can be then shown that the eigen-vector of $\mathbf{C}$ corresponding to its one non-zero eigenvalue is of the form

$$[1, \exp(j\gamma), \exp(2j\gamma), \exp(3j\gamma)]^T/2 \tag{26}$$

where $\exp(j\gamma) = \frac{\bar{a}}{|a|}$ so that $p = q = 1/2$. The choice of r, s can be arbitrary (subject to the norm constraint) since the associated eigen-value is zero.