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1 Introduction

In this contribution we consider precoding codebook design for the 4TX downlink and detail a codebook structure that is suitable for both the uniform linear array (ULA) and the cross-pole antenna configurations. The codebook structure is derived using fundamental properties of the spatial correlation matrices under the ULA and cross-pole antenna configurations. Each precoding codeword is derived as the product of two matrices, as in the 8 TX case. In the following, unless otherwise mentioned we will assume the co-polarized antennas to be closely spaced.

2 Uniform Linear Array

We have the following observations for the uniform linear array (ULA) transmit antenna configuration. Consider a system with N co-polarized transmit antennas and let $\mathbf{C}$ denote the transmit spatial correlation matrix. Let us define $\mathbf{J}$ to be the matrix which has zeros everywhere except on the cross diagonal elements, i.e., $\mathbf{J} = [J_{m,n}]$ where

$$J_{m,n} = \begin{cases} 1, & n = N - m + 1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

A vector is said to be Hermitian if

$$\bar{\mathbf{x}} = \mathbf{J}\mathbf{x}, \tag{2}$$

where $\bar{\mathbf{x}}$ denotes the conjugate of $\mathbf{x}$. We offer the following set of properties:

Lemma 1 *The matrix* $\mathbf{C}$ *is a* Hermitian Toeplitz *matrix, i.e.,* $\mathbf{C}$ *satisfies*

$$\bar{\mathbf{C}} = \mathbf{J}\mathbf{C}\mathbf{J}, \tag{3}$$

where $\bar{\mathbf{C}}$ *denotes the conjugate of* $\mathbf{C}$.

Lemma 2 *The eigenspace of any Hermitian Toeplitz matrix can be completely described by Hermitian vectors. In other words, given a Hermitian Toeplitz matrix* $\mathbf{A}$ *and with* $\mathbf{x}$ *being its eigenvector such that* $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$*, then* $\exists\, \mathbf{y}$ *such that* $\bar{\mathbf{y}} = \mathbf{J}\mathbf{y}$ *with*

$$\mathbf{A}\mathbf{y} = \lambda\mathbf{y} \tag{4}$$

Lemma 3 *Suppose* λ *is an eigenvalue of a Hermitian Toeplitz matrix* $\mathbf{A}$ *with algebraic multiplicity one. Then if* $\mathbf{x}$ *is an eigenvector such that* $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$*, we must have*

$$\mathbf{J}\mathbf{x} = \exp(j\delta)\bar{\mathbf{x}}. \tag{5}$$

for some $\delta \in [0, 2\pi)$ *and where* $j = \sqrt{-1}$.

A simplified model for the correlation matrix is the exponential correlation model, which is discussed further in the Appendix, and is given by

$$\mathbf{C} = [C_{m,n}]_{m,n=1}^{N},\ C_{m,n} = \rho^{|m-n|}\exp(j\theta(m-n)),\ m, n \in \{1, \cdots, N\}, \tag{6}$$

where $\rho \in [0, 1]$ & $\theta \in [0, 2\pi)$.

2.1 4 TX ULA

In this section, we consider the case of $N = 4$ co-polarized transmit antennas. First, note that without loss of generality, we can impose the following structure on each eigenvector $\mathbf{x}$ of the spatial correlation matrix $\mathbf{C}$,

$$\mathbf{x} = [a_1, a_2\exp(j\theta_2), a_3\exp(j\theta_3), a_4\exp(j\theta_4)]^T, \tag{7}$$

where $a_1, a_2, a_3, a_4 \in \mathbb{R}_+$. Recalling that the matrix $\mathbf{C}$ must be a Hermitian Toeplitz matrix and invoking Lemmas 2 and 3, we can deduce that we must have

$$\begin{aligned} a_1 = a_4,\ a_2 = a_3 \\ \theta_2 + \theta_3 = \theta_4. \end{aligned} \tag{8}$$

Then, consider any two eigen-vectors of the form in (8) given by

$$\mathbf{x} = [a, b\exp(j\theta_2), b\exp(j\theta_3), a\exp(j(\theta_2+\theta_3))]^T \tag{9}$$

$$\mathbf{y} = [c, d\exp(j\gamma_2), d\exp(j\gamma_3), c\exp(j(\gamma_2+\gamma_3))]^T, \tag{10}$$

where $a, b, c, d \in I\!R_+$. Then, a sufficient condition to enforce orthogonality among these two eigenvectors is to ensure that

$$\begin{aligned} \theta_2 + \theta_3 &= \pm\pi + \gamma_2 + \gamma_3 \\ \theta_2 - \theta_3 &= \pm\pi + \gamma_2 - \gamma_3 \end{aligned}$$

which can be simplified to

$$\begin{aligned} \theta_2 &= t\pi + \gamma_2, t \in \{0, \pm 1\} \\ \theta_3 &= \pm(1-|t|)\pi + \gamma_3. \end{aligned} \tag{11}$$

We remark that (11) is not necessary but works for all possible values of the scalars $a, b, c, d \in I\!R_+$.

3 Polarized Setup

Suppose the transmitter has $2N$ cross-polarized antennas comprising of a pair of N co-polarized antennas each. Then the correlation matrix of each one of these two co-polarized sets is denoted by $\mathbf{C}$ which is Hermitian and Toeplitz. The overall $2N \times 2N$ correlation matrix $\tilde{\mathbf{C}}$ can be written as

$$\tilde{\mathbf{C}} = \begin{bmatrix} 1 & \alpha \\ \bar{\alpha} & 1 \end{bmatrix} \otimes \mathbf{C} \tag{12}$$

where $\otimes$ denotes the kronecker product and $\alpha \in \mathbb{C} : |\alpha| \in [0, 1]$. It can be shown that any eigenvector $\tilde{\mathbf{y}}$ of $\tilde{\mathbf{C}}$ has the form

$$\tilde{\mathbf{y}} = \mathbf{y} \otimes \mathbf{x} \tag{13}$$

where $\mathbf{y} \in \mathbb{C}^{2\times 1}$ is an eigenvector of the matrix $\begin{bmatrix} 1 & \alpha \\ \bar{\alpha} & 1 \end{bmatrix}$ and $\mathbf{x}$ is an eigenvector of $\mathbf{C}$. Furthermore, the two eigenvectors of the matrix $\begin{bmatrix} 1 & \alpha \\ \bar{\alpha} & 1 \end{bmatrix}$ are $\frac{\exp(j\beta)}{\sqrt{2}}[1 \quad \frac{\alpha}{|\alpha|}]^T$ and $\frac{\exp(j\beta)}{\sqrt{2}}[1 \quad -\frac{\alpha}{|\alpha|}]^T$, where $(.)^T$ denotes the transpose operation and the phase term $\exp(j\beta)$ can be ignored without loss of optimality. The two eigenvalues are $1 \pm |\alpha|$. Note that the matrix $\begin{bmatrix} 1 & \alpha \\ \bar{\alpha} & 1 \end{bmatrix}$ also models the correlation matrix of the 2 transmit ULA.

4 Codebook Construction in product form

We next discuss a codebook construction in which each codeword is derived as a matrix product. We use the codebook designed in [1] (by downsampling the 8TX codebook) as the base and expand it while conforming to the principles outlined in Sections 2 and 3. Let $\mathbf{w}_n = [1 \ \exp(j2\pi n/16)]^T$ for $n = 0, \cdots, 15$. We define the inner (wideband) codebook as

$$\mathcal{C}^{(1)} = \left\{ \begin{bmatrix} \mathbf{A}(q) \odot \mathbf{W}^{(1)}(k) & \mathbf{0} \\ \mathbf{0} & \mathbf{B}(q) \odot \mathbf{W}^{(1)}(k) \end{bmatrix} : \right.$$
$$\left. \mathbf{W}^{(1)}(k) = [\mathbf{w}_{2k \bmod 16}, \mathbf{w}_{2k+1 \bmod 16}, \mathbf{w}_{2k+2 \bmod 16}, \mathbf{w}_{2k+3 \bmod 16}], \ k = 0, \cdots, 7; \ q = 1, \cdots, Q \right\}, \tag{14}$$

where $\odot$ denotes Hadamard product and

$$\begin{aligned} \mathbf{A}(q) &= \begin{bmatrix} 1 & 0 \\ 0 & \exp(j2\pi\, d_q) \end{bmatrix} \begin{bmatrix} a_q & a_q & b_q & b_q \\ b_q & b_q & a_q & a_q \end{bmatrix} \\ \mathbf{B}(q) &= \exp(j2\pi\gamma_q) \begin{bmatrix} 1 & 0 \\ 0 & \exp(j2\pi\, d_q) \end{bmatrix} \begin{bmatrix} b_q & b_q & a_q & a_q \\ a_q & a_q & b_q & b_q \end{bmatrix}, \quad q = 1, \cdots, Q, \end{aligned} \tag{15}$$

where $\gamma_q, a_q, b_q, d_q \in [0, 1]$ & $a_q^2 + b_q^2 = 1/2 \ \forall \ q$. The rank-1 outer codebook is defined as

$$\mathcal{C}_1^{(2)} = \left\{ \begin{bmatrix} \mathbf{y} \\ \mathbf{y} \end{bmatrix}, \begin{bmatrix} \mathbf{y} \\ -\mathbf{y} \end{bmatrix}, \begin{bmatrix} \mathbf{y} \\ j\mathbf{y} \end{bmatrix}, \begin{bmatrix} \mathbf{y} \\ -j\mathbf{y} \end{bmatrix} \right\} \tag{16}$$

$$\mathbf{y} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}, \tag{17}$$

where $\mathbf{e}_i$ denotes the 4×1 column selection vector. The outer rank-2 codebook is defined as

$$\mathcal{C}_2^{(2)} = \left\{ \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 \\ \mathbf{y}_1 & -\mathbf{y}_2 \end{bmatrix}, \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 \\ j\mathbf{y}_1 & -j\mathbf{y}_2 \end{bmatrix} \right\}, \tag{18}$$

$$(\mathbf{y}_1, \mathbf{y}_2) = \{(\mathbf{e}_1, \mathbf{e}_1), (\mathbf{e}_2, \mathbf{e}_2), (\mathbf{e}_3, \mathbf{e}_3), (\mathbf{e}_4, \mathbf{e}_4), (\mathbf{e}_1, \mathbf{e}_3), (\mathbf{e}_1, \mathbf{e}_4), (\mathbf{e}_2, \mathbf{e}_3), (\mathbf{e}_2, \mathbf{e}_4)\} \tag{19}$$

We note that one choice of γ_q, d_q is to set $d_q = \gamma_q/2 = \theta_q$ for some configurable scalars $\{\theta_q \in [0, 1]\}$. Under this choice, we next describe a way of determining a set of triplets $\{\theta_q, a_q, b_q\}_{q=1}^{Q}$. From the discussion in Appendix 6 using the exponential correlation model, we relate $\exp(j2\pi\theta) = \frac{\bar{a}}{|a|}$ so that a good choice is to assume that the (un-quantized) θ is uniformly distributed in $[0, 1)$. Thus, a good strategy to obtain a finite set $\Theta = \{\theta\}$ is via uniform quantization of $[0, 1)$ using the given number of bits. Considering now the selection of a_q, b_q, one possibility is to relate them to the variables p, q discussed in Appendix 6. Accordingly, a finite set of values for the correlation magnitude parameter $\rho = |a|$ can be selected from which a set of vectors $\mathcal{S} = \{[p, q]\}$ can be obtained after invoking the formulas in Appendix 6. Then, the set of triplets $\{\theta_q, a_q, b_q\}$ can be defined as the Cartesian product $\Theta \otimes \mathcal{S}$, where we have used $\otimes$ to denote the Cartesian product, whose size is $Q = |\Theta||\mathcal{S}|$.

The rank-3 and rank-4 codebooks can be fixed to the legacy (Householder) rank-3 and rank-4 codebooks. In addition, the entire legacy codebook can be included as a subset.

We note that not all codeword matrices in the aforementioned codebook satisfy the constant magnitude property. However, the codebook has the desirable property that for each rank and for each choice of the inner (wideband) precoder, the expected value of each row norm square (i.e., sum of the magnitude squares of elements in that row) of the final selected precoder matrix is identical, where the expectation is computed over the set of outer (subband) precoders assuming that they are chosen equi-probably.

5 Conclusions

In this contribution we detailed a codebook structure and presented an embodiment which conforms to the matrix product form. This structure is motivated by fundamental properties of the spatial correlation matrix and makes codebook optimization feasible. *However, an enhanced codebook must be considered only if sufficient gain is observed over the existing Householder codebook.* In addition, it is better to focus on such enhancement after investigating related issues in the 3D MIMO channel.

References

[1] Ericsson,ST-Ericsson, "Design and Evaluation of 4 TX Precoder Codebooks for CSI Feedback," *3GPP TSG RAN WG1 R1-104847* 62, Madrid, August 2010.

6 Appendix: 4 TX ULA with the exponential correlation model

Next, we consider the case where we further specialize the correlation matrix to be

$$\mathbf{C} = \begin{bmatrix} 1 & a & a^2 & a^3 \\ b & 1 & a & a^2 \\ b^2 & b & 1 & a \\ b^3 & b^2 & b & 1 \end{bmatrix}, \tag{20}$$

where $a \in \mathbb{C}$ such that $|a| \leq 1$ and $b = \bar{a}$. Note that the matrix $\mathbf{C}$ is Hermitian Toeplitz and is also completely characterized by one complex scalar. Thus its eigenvectors can be expected to have more structure in addition to that possessed by an eigenvector of a general Hermitian Toeplitz matrix. We will also exploit this additional structure in the following. The matrix $\mathbf{J}$ for this case can be written as,

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

We first consider the case $|a| < 1$. In this case, the eigen-vectors of any matrix of the form in (20) have the following properties. Consider any matrix $\mathbf{C}$ of the form in (20) and let

$$\mathbf{C} = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^{\dagger} \tag{21}$$

denote its eigen-decomposition where $(.)^{\dagger}$ denotes the conjugate transpose operation and $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ with $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ denoting the four real-valued eigenvalues. Then,

$$\mathbf{E} = \mathbf{D}_p(\mathbf{H} \odot \mathbf{S}) \tag{22}$$

where $\odot$ denotes the Hadamard product and $\mathbf{D}_p$ is a diagonal matrix of the form

$$\mathbf{D}_p = \text{diag}\{1, \exp(j\gamma), \exp(2j\gamma), \exp(3j\gamma)\} \tag{23}$$

for some $\gamma \in [0, 2\pi)$. The matrix $\mathbf{S}$ has the following structure

$$\mathbf{S} = \begin{bmatrix} p & r & q & s \\ q & s & p & r \\ q & s & p & r \\ p & r & q & s \end{bmatrix} \tag{24}$$

for some real positive scalars p, q, r, s such that $q = \sqrt{1/2 - p^2}$ and $r = \sqrt{1/2 - s^2}$. The matrix $\mathbf{H}$ is a 4×4 real-valued Hadamard matrix, i.e., columns of $\mathbf{H}$ are mutually orthogonal and all its elements belong to the set $\{\pm 1\}$. Then, since each column of $\mathbf{E}$ must satisfy the condition in (5), each column of $\mathbf{H} = [\mathbf{h}_1, \cdots \mathbf{h}_4]$ must satisfy the following conditions.

$$h_{1i}h_{4i} = h_{2i}h_{3i}, \ \forall \ i = 1, 2, 3, 4. \tag{25}$$

Also, since $\mathbf{E}$ must be a unitary matrix, $\mathbf{H}$ must also satisfy the following additional conditions.

$$h_{11}h_{12} = -h_{41}h_{42}, h_{21}h_{22} = -h_{31}h_{32}; h_{11}h_{14} = -h_{41}h_{44}, h_{21}h_{24} = -h_{31}h_{34}$$
$$h_{22}h_{23} = -h_{32}h_{33}, h_{12}h_{13} = -h_{42}h_{43};$$

An important example $\mathbf{H}$ is the following:

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}$$

Using $\mathbf{H}$ given above and recalling that $\rho = |a|$, we can derive formulas that yield the scalars p, q, r, s as follows. First we have that $\exp(j\gamma) = \frac{\bar{a}}{|a|}$. Substituting this in (22) yields after some manipulations that

$$q = \frac{\rho(1+\rho)}{\sqrt{2[\rho^2(1+\rho)^2 + (\vartheta - \rho)^2]}}, \tag{26}$$

where $\vartheta = \frac{\rho+\rho^3+\sqrt{\rho^2(1+\rho^2)^2+4\rho^2(1+2\rho)}}{2}$ and $p = \sqrt{1/2 - q^2}$. Note that in the special case when $\rho = 0$ the correlation matrix $\mathbf{C}$ reduces to the identity matrix so that we can chose p, q, r, s arbitrarily (subject to the respective norm constraints). Further, for $\rho < 1$, we can determine that $r = \sqrt{\frac{\zeta+\rho}{2(2\zeta+\rho+\rho^3)}}$ with $s = \sqrt{1/2 - r^2}$ where $\zeta = \frac{-\rho-\rho^3+\sqrt{(\rho-\rho^3)^2+4\rho^2(1-\rho)^2}}{2}$.

On the other hand when $\rho = 1$ we note that the matrix $\mathbf{C}$ is a rank-1 matrix that is given by

$$\mathbf{C} = \begin{bmatrix} 1 \\ b \\ b^2 \\ b^3 \end{bmatrix} \begin{bmatrix} 1 & a & a^2 & a^3 \end{bmatrix}. \tag{27}$$

It can be then shown that the eigen-vector of $\mathbf{C}$ corresponding to its one non-zero eigenvalue is of the form

$$[1, \exp(j\gamma), \exp(2j\gamma), \exp(3j\gamma)]^T/2 \tag{28}$$

where $\exp(j\gamma) = \frac{\bar{a}}{|a|}$ so that $p = q = 1/2$. The choice of r, s can be arbitrary (subject to the norm constraint) since the associated eigen-value is zero.