

---

**Agenda Item:** 6.3.4  
**Source:** NEC Group  
**Title:** MU-MIMO: CQI Computation and PMI Selection  
**Document for:** Discussion and Decision

---

## 1 Introduction

We consider a multi-user multi-input-multi-output (MU-MIMO) downlink channel where the eNodeB may schedule several user terminals on the same sub-band. As per the dimensioning agreement [1], each scheduled user is served a single data stream using beamforming or two data streams using rank-2 precoding. Each active user reports a preferred matrix index (PMI) (per sub-band) to the eNodeB, which is an index that identifies either a particular vector in a codebook of unit norm vectors or a particular matrix in a codebook of semi-unitary matrices. In addition, as per the double codebook structure the user can also report a wideband PMI [1]. The codebooks are known in advance to the eNodeB as well as all users. Each user also reports up-to two channel quality indices (CQIs) (per sub-band) which are its estimates of the signal-to-interference-plus-noise ratios (SINRs) on that sub-band. The reported PMIs and CQIs are then employed by the eNodeB to determine a suitable set of scheduled users, their transmit precoders and their assigned rates. The key practical problem in MU-MIMO is that when computing its PMI and CQIs, a user does not have an accurate estimate of the interference it might see (if scheduled) from the signals intended for the other co-scheduled users. This results in a mismatch between the user reported SINRs and the ones it actually observes in the aftermath of scheduling. In [2] it is shown that reducing this mismatch is critical for achieving MU-MIMO gains.

*To alleviate this problem we consider informing each user (in a slow or semi-static manner) about an estimate of (or an upper bound on) the total number of streams that the eNodeB*

*expects to co-schedule on a sub-band.* Such total number of streams can be fixed for any sub-band, meaning that only a single signalling is needed for the whole bandwidth. A related proposal [6] suggests reference rank indication for improving the accuracy of CQI computation. In addition, the indication of the expected total number of streams to a user can also be accompanied by a suggested rank for that user. We recall that slow rank adaptation was also recommended in [7] for MU-MIMO. Therefore in [10], we have suggested useful SINR formulas that can use such parameters and have shown that the mismatch problem is mitigated to a large extent. We have also considered best-companion PMI feedback in [10] that was suggested in [9] to reduce the post-scheduling interference seen by the user.

In this document, we provide some system level simulation results and more details on the proof and derivations for the proposed formulas given in [10].

## 2 PMI Selection and CQI computation Rules

Consider the narrowband received signal model at a user terminal of interest that is equipped with  $N$  receive antennas and where the eNodeB has  $M$  transmit antennas,

$$\mathbf{y} = \mathbf{H}^\dagger \mathbf{x} + \boldsymbol{\eta}, \quad (1)$$

where  $\mathbf{H} \in \mathbb{C}^{M \times N}$  is the channel matrix and  $\boldsymbol{\eta} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$  is the additive noise. The signal vector  $\mathbf{x}$  transmitted by the eNodeB can be expanded as

$$\mathbf{x} = \sum_{k \in \mathcal{U}} \mathbf{V}_k \mathbf{s}_k \quad (2)$$

where  $\mathcal{U}$  is the set of users that are scheduled.  $\mathbf{V}_k$  is an  $M \times r_k$  semi-unitary matrix with  $\mathbf{V}_k^\dagger \mathbf{V}_k = \mathbf{I}_{r_k}$  and  $\mathbf{s}_k$  is the  $r_k \times 1$  symbol vector corresponding to user  $k \in \mathcal{U}$ . Further, let  $S = \sum_{k \in \mathcal{U}} r_k$  be the total number of streams that are co-scheduled. Each scheduled stream is assigned an identical power  $\rho'$ . The set of precoding matrices or vectors  $\{\mathbf{V}_k\}_{k \in \mathcal{U}}$  are

determined by first selecting a set  $\mathcal{U}$  of users that have reported mutually (near-)orthogonal matrices or vectors  $\{\mathbf{G}_k\}_{k \in \mathcal{U}}$  and which also yield a high weighted sum rate. The eNodeB can then either set  $\mathbf{V}_k = \mathbf{G}_k \forall k \in \mathcal{U}$  or it can perform a modified version of the block diagonalization technique proposed in [3] on  $\{\mathbf{G}_k\}_{k \in \mathcal{U}}$  to determine  $\{\mathbf{V}_k\}_{k \in \mathcal{U}}$ .

The user of interest can estimate  $\rho' \mathbf{H}$ . We assume that the user of interest also has an estimate of the expected total number of streams,  $S$ , available. In practise, the eNodeB can convey an estimate of  $S$  (or an upper bound on  $S$ ) to a user in a semi-static manner and such an estimate can be user-specific. The user then uses its estimates of  $\rho' \mathbf{H}$  and  $S$  to select PMIs and compute SINRs with the formulas provided in [10], which we recap in Appendix A. The detail derivations of the formulas for PMI selections are provided in Appendix B.

### 3 Simulation Results

In this section, we present some simulation results using the proposed PMI selection and SINR computations in [10] and in Appendix A as well. The settings for system level simulations are provided in Table 1. We follow the assumptions for MU-MIMO system-level simulations that were agreed in [12], i.e., maximum 4 co-scheduled layers per RB and maximum 2 layers per UE. Due to extremely high complexity for optimal UE pairing, we also introduce an additional constraint that is maximum 2 UEs pairing for every sub-band in our simulations. For all simulations, we use the Rel. 8 codebooks for PMI feedbacks.

Table 2 and Table 3 show the average cell spectral efficiency and 5% cell edge spectral efficiency of SU-MIMO using proportional fair scheduling with sub-band size 1RB and 5RB, respectively. With the proposed feedback schemes, the average cell spectral efficiency performance for MU-MIMO is provided in Table 4 with various values of the expected total number of streams  $S$ , i.e.,  $S = 2, 3, 4$ , which is fixed for the whole bandwidth. The maximum sum-rate scheduler is employed in these simulations. We find that for  $4 \times 2$  MU-MIMO,  $S = 3$  provides the best performance.

We now evaluate the performance of MU-MIMO with proposed PMI selection and CQI feedback using the proportional fair scheduling and compare it with that of the SU-MIMO system. The spectral efficiency results are shown in Table 5 and Table 6 for the subband size of 1RB and 5RB, respectively. We can see that compared with the SU-MIMO performance in Table 2 and Table 3, a certain performance gains are observed for MU-MIMO over the SU-MIMO. In particular, for the sub-band size of 1RB, the gains of MU-MIMO over the corresponding SU-MIMO cases are 8.7% on the average cell spectral efficiency and 18.7% on the 5% cell edge spectral efficiency, respectively. For the sub-band size of 5RB, the gain over SU-MIMO case is 8.5% on the the average cell spectral efficiency and 8.1% on the 5% cell edge spectral efficiency. These results demonstrate that indicating the expected total number of streams to a user realizes the benefits of MU-MIMO.

## 4 Conclusions

In this contribution, we considered enhancements to the MU-MIMO operation which aim to reduce the mismatch between the user reported SINRs and the ones it actually observes in the aftermath of scheduling. We provided some details on the derivation of the proposed scheme on PMI selection and CQI computation. We also provided some system-level simulation results for SU-MIMO and MU-MIMO with proposed feedback. The simulation results show that indicating the expected total number of streams (that the eNodeB expects to co-schedule on a sub-band) to a user indeed realize the benefits of MU-MIMO. In addition, the indication of the expected total number of streams to a user can also be accompanied by a suggested rank for that user.

## Appendix A PMI Selection and SINR computation

We first consider the case when the user reports one PMI along with one or more CQIs per sub-band (each based on a computed SINR). In general the PMI can correspond to a precoder

of rank  $r$ , where  $1 \leq r \leq \min\{M, N\}$ . In the scenario where fast-rank adaptation is allowed in MU-MIMO, the user can determine the best precoder for each value of  $r$  and then select the one which yields the overall highest rate. In the case where only slow rank adaptation is allowed, the user selects precoder matrices of a common rank for several consecutive frames and the eNodeB can possibly inform the user about a suitable rank in a semi-static manner.

Next, in order to determine a suitable semi-unitary matrix  $\hat{\mathbf{G}}_r$  from a set or codebook of rank- $r$  semi-unitary matrices,  $\mathcal{C}_r$ , along with up-to  $r$  SINRs, the user of interest can use the following rules. The key feature of the rules derived below is that they attempt to account for the interference due to signals intended for the co-scheduled users and maximize a bound on the expected SINR or the expected capacity. These rules are derived by leveraging some key random matrix distribution results developed in [3]. Note that the codebook of semi-unitary matrices could itself be obtained after transformation of a base codebook [8].

- Capacity Metric: The PMI is selected as follows:

$$\begin{aligned} \hat{\mathbf{G}}_r &= \arg \max_{\mathbf{G} \in \mathcal{C}_r} \left\{ \ln \left| \mathbf{I} + \rho' \mathbf{G}^\dagger \mathbf{H} (\mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger (\mathbf{I} - \mathbf{G} \mathbf{G}^\dagger) \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{G} \right| \right\} \\ &= \arg \max_{\mathbf{G} \in \mathcal{C}_r} \left\{ \ln \left| \mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger (\mathbf{I} - \mathbf{G} \mathbf{G}^\dagger) \mathbf{H} + \rho' \mathbf{H}^\dagger \mathbf{G} \mathbf{G}^\dagger \mathbf{H} \right| - \ln \left| \mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger (\mathbf{I} - \mathbf{G} \mathbf{G}^\dagger) \mathbf{H} \right| \right\} \end{aligned} \quad (3)$$

where  $\tilde{\rho} = \frac{\rho'(S-r)}{M-r}$ .

- MMSE based receiver: A PMI is selected after determining  $r$  SINRs for each matrix in  $\mathcal{C}_r$ . We let  $\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_r]$ .

$$\hat{\mathbf{G}}_r = \arg \max_{\mathbf{G} \in \mathcal{C}_r} \left\{ \sum_{j=1}^r \ln (1 + \text{SINR}_{j,r}^{\text{MMSE}}(\mathbf{G})) \right\} \quad (4)$$

where

$$\text{SINR}_{j,r}^{\text{MMSE}}(\mathbf{G}) = \frac{\rho' \mathbf{g}_j^\dagger \mathbf{H} (\mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger \mathbf{H} + (\rho' - \tilde{\rho}) \mathbf{H}^\dagger \mathbf{G} \mathbf{G}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{g}_j}{1 - \rho' \mathbf{g}_j^\dagger \mathbf{H} (\mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger \mathbf{H} + (\rho' - \tilde{\rho}) \mathbf{H}^\dagger \mathbf{G} \mathbf{G}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{g}_j} \quad (5)$$

- SIC receiver: A PMI is selected after determining  $r$  SINRs for each matrix in  $\mathcal{C}_r$ . We let  $\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_r]$  and suppose the order of decoding to be  $\{1, \dots, r\}$ .

$$\hat{\mathbf{G}}_r = \arg \max_{\mathbf{G} \in \mathcal{C}_r} \left\{ \sum_{j=1}^r \ln (1 + \text{SINR}_{j,r}^{\text{SIC}}(\mathbf{G})) \right\} \quad (6)$$

where

$$\text{SINR}_{j,r}^{\text{SIC}}(\mathbf{G}) = \frac{\rho' \mathbf{g}_j^\dagger \mathbf{H} \left( \mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger (\mathbf{I} - \mathbf{G} \mathbf{G}^\dagger) \mathbf{H} + \rho' \mathbf{H}^\dagger \sum_{q=j}^r \mathbf{g}_q \mathbf{g}_q^\dagger \mathbf{H} \right)^{-1} \mathbf{H}^\dagger \mathbf{g}_j}{1 - \rho' \mathbf{g}_j^\dagger \mathbf{H} \left( \mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger (\mathbf{I} - \mathbf{G} \mathbf{G}^\dagger) \mathbf{H} + \rho' \mathbf{H}^\dagger \sum_{q=j}^r \mathbf{g}_q \mathbf{g}_q^\dagger \mathbf{H} \right)^{-1} \mathbf{H}^\dagger \mathbf{g}_j} \quad (7)$$

Note that using the chain rule

$$\begin{aligned} \sum_{j=1}^r \ln (1 + \text{SINR}_{j,r}^{\text{SIC}}(\mathbf{G})) &= \ln |\mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger (\mathbf{I} - \mathbf{G} \mathbf{G}^\dagger) \mathbf{H} + \rho' \mathbf{H}^\dagger \mathbf{G} \mathbf{G}^\dagger \mathbf{H}| \\ &\quad - \ln |\mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger (\mathbf{I} - \mathbf{G} \mathbf{G}^\dagger) \mathbf{H}| \end{aligned}$$

so that an optimal PMI can be computed using (3) and then  $r$  SINRs can be computed for the  $\hat{\mathbf{G}}_r$  so determined.

- Single unified SINR for MMSE receiver when  $r > 1$ : A PMI is selected after determining one SINR for each matrix in  $\mathcal{C}_r$ . We let  $\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_r]$ .

$$\hat{\mathbf{G}}_r = \arg \max_{\mathbf{G} \in \mathcal{C}_r} \left\{ \ln (1 + \text{SINR}_r^{\text{MMSE}}(\mathbf{G})) \right\} \quad (8)$$

where

$$\begin{aligned} \ln (1 + \text{SINR}_r^{\text{MMSE}}(\mathbf{G})) &= \ln |\mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger \mathbf{H} + (\rho' - \tilde{\rho}) \mathbf{H}^\dagger \mathbf{G} \mathbf{G}^\dagger \mathbf{H}| \\ &\quad - \ln \left| \mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger \mathbf{H} + \left( \frac{\rho'(r-1)}{r} - \tilde{\rho} \right) \mathbf{H}^\dagger \mathbf{G} \mathbf{G}^\dagger \mathbf{H} \right| \end{aligned}$$

The following two special cases are particularly important

- Rank-1 or beamforming with  $r = 1$ . In this case an optimal vector  $\hat{\mathbf{g}}_1 \in \mathcal{C}_1$  is selected and only one SINR (per vector) is computed. Specializing either (4) or (6) to this case, we have

$$\hat{\mathbf{g}}_1 = \arg \max_{\mathbf{g} \in \mathcal{C}_1} \{ \text{SINR}_{1,1}^{\text{MMSE}}(\mathbf{g}) \} \quad (9)$$

where

$$\text{SINR}_{1,1}^{\text{MMSE}}(\mathbf{g}) = \rho' \mathbf{g}^\dagger \mathbf{H} (\mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger (\mathbf{I} - \mathbf{g} \mathbf{g}^\dagger) \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{g}$$

with  $\tilde{\rho} = \frac{\rho'(S-1)}{M-1}$ . The rule given in (9) was derived in [4] and was shown to be very effective in [5]. It can be shown that the rule in (9) is equivalent to the following rule that is much simpler to compute.

$$\hat{\mathbf{g}}_1 = \arg \max_{\mathbf{g} \in \mathcal{C}_1} \{ \mathbf{g}^\dagger \mathbf{H} (\mathbf{I} + \tilde{\rho} \mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{g} \} \quad (10)$$

- Rank-2 or precoding with  $r = 2$ . In this case an optimal matrix  $\hat{\mathbf{G}}_2 \in \mathcal{C}_2$  can be selected using the SIC formula in (6) with  $r = 2$  and 2 CQIs based on the computed SINRs can be fed back in the form of a base-CQI and a delta-CQI. Expanding  $\hat{\mathbf{G}}_2 = [\hat{\mathbf{g}}_{1,2}, \hat{\mathbf{g}}_{2,2}]$ , we note that in this case the CQI computed using first SINR,  $\text{SINR}_{1,2}^{\text{SIC}}(\hat{\mathbf{G}}_2)$ , is the base CQI. The second CQI is equal to the base-CQI plus delta-CQI and corresponds to the second SINR,  $\text{SINR}_{2,2}^{\text{SIC}}(\hat{\mathbf{G}}_2)$ . This allows the eNodeB to perform a rank-override in which the user is scheduled as a rank-1 MU-MIMO user based on the pair  $(\text{SINR}_{2,2}^{\text{SIC}}, \hat{\mathbf{g}}_{2,2})$ .

Alternatively, an optimal matrix  $\hat{\mathbf{G}}_2 \in \mathcal{C}_2$  can be selected using the unified formula in (8) with  $r = 2$ . The base CQI can correspond to  $\text{SINR}_2^{\text{MMSE}}(\hat{\mathbf{G}}_2)$ . The second CQI is

equal to the base-CQI plus delta-CQI and corresponds to the following SINR

$$\frac{\rho' \hat{\mathbf{g}}_{2,2}^\dagger \mathbf{H} (\mathbf{I} + \hat{\rho} \mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \hat{\mathbf{g}}_{2,2}}{1 - \hat{\rho} \hat{\mathbf{g}}_{2,2}^\dagger \mathbf{H} (\mathbf{I} + \hat{\rho} \mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \hat{\mathbf{g}}_{2,2}} \quad (11)$$

with  $\hat{\rho} = \frac{\rho'(S-1)}{M-1}$ . This allows the eNodeB to perform a rank-override in which the user is scheduled as a rank-1 MU-MIMO user based on  $\hat{\mathbf{g}}_{2,2}$  and the SINR given in (11).

### Alternate PMI Selection and CQI computation Rules

We now consider two alternate rules.

- The user can first select a matrix from  $\mathcal{C}_r$  by quantizing the  $r$  dominant right singular vectors of  $\mathbf{H}^\dagger$ . In particular, let  $\mathbf{H}^\dagger = \mathbf{U} \mathbf{\Lambda} \tilde{\mathbf{V}}^\dagger$  be the SVD of  $\mathbf{H}^\dagger$  where  $\tilde{\mathbf{V}}$  is a  $M \times N$  semi-unitary matrix. Let  $\tilde{\mathbf{V}}_{(r)}$  denote the matrix formed by the first  $r$  columns of  $\tilde{\mathbf{V}}$  which are the  $r$  dominant right singular vectors of  $\mathbf{H}^\dagger$ . Then, the user can select a matrix from  $\mathcal{C}_r$  by using

$$\hat{\mathbf{G}}_r = \arg \max_{\mathbf{G} \in \mathcal{C}_r} \left\{ \text{tr}(\tilde{\mathbf{V}}_{(r)}^\dagger \mathbf{G}_r \mathbf{G}_r^\dagger \tilde{\mathbf{V}}_{(r)}) \right\} \quad (12)$$

Once such a  $\hat{\mathbf{G}}_r$  is determined we can compute the  $r$  SINRs corresponding to  $\hat{\mathbf{G}}_r$  using either (5) or (7).

- SU-MIMO Based Rule: To determine a suitable precoding matrix from  $\mathcal{C}_r$  along with corresponding SINRs, the user of interest can use the following SU-MIMO rules which completely neglect the interference that will be caused due to other co-scheduled streams. For convenience, we only consider the MMSE based receiver. Then, a PMI is selected after determining  $r$  SINRs for each matrix in  $\mathcal{C}_r$ . We let  $\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_r]$ .

$$\hat{\mathbf{G}}_r = \arg \max_{\mathbf{G} \in \mathcal{C}_r} \left\{ \sum_{j=1}^r \ln(1 + \text{SINR}_{j,r}^{\text{MMSE}}(\mathbf{G})) \right\} \quad (13)$$

where

$$\text{SINR}_{j,r}^{\text{MMSE}}(\mathbf{G}) = \frac{\rho}{r} \mathbf{g}_j^\dagger \mathbf{H} (\mathbf{I} + \frac{\rho}{r} \mathbf{H}^\dagger \mathbf{G} \mathbf{G}^\dagger \mathbf{H} - \frac{\rho}{r} \mathbf{H}^\dagger \mathbf{g}_j \mathbf{g}_j^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{g}_j$$

with  $\rho$  denoting the total power that is equally divided among all streams.

### PMI, Best-Companion-PMI Selection and CQI computation

The user of interest must now feed-back to its serving eNodeB, a PMI along with a best-companion PMI, with the understanding that the codeword corresponding to such a companion PMI generates the least amount of interference for the user. In addition to  $S$  (and possibly  $r$ ), we assume that the user of interest knows (or has been informed about)  $r^c$ , the rank of the codeword to be selected as the companion.

- Capacity Metric: a matrix  $\hat{\mathbf{G}}_r$  is selected along with the best-companion matrix,  $\hat{\mathbf{G}}_{r^c}$ .

$$\{\hat{\mathbf{G}}_r, \hat{\mathbf{G}}_{r^c}\} = \arg \max_{\mathbf{G} \in \mathcal{C}_r, \mathbf{G}_{r^c} \in \mathcal{C}_{r^c}} \left\{ \ln \left| \mathbf{I} + \check{\rho} \mathbf{H}^\dagger \mathbf{P}_{\mathbf{G}, \mathbf{G}_{r^c}}^\perp \mathbf{H} + \rho' \mathbf{H}^\dagger \mathbf{G} \mathbf{G}^\dagger \mathbf{H} + \rho' \mathbf{H}^\dagger \mathbf{G}_{r^c} \mathbf{G}_{r^c}^\dagger \mathbf{H} \right| \right. \\ \left. - \ln \left| \mathbf{I} + \check{\rho} \mathbf{H}^\dagger \mathbf{P}_{\mathbf{G}, \mathbf{G}_{r^c}}^\perp \mathbf{H} + \rho' \mathbf{H}^\dagger \mathbf{G}_{r^c} \mathbf{G}_{r^c}^\dagger \mathbf{H} \right| \right\}$$

where  $\check{\rho} = \frac{\rho'(S-r-r^c)}{r'}$ ,  $\mathbf{P}_{\mathbf{G}, \mathbf{G}_{r^c}}^\perp = \mathbf{I} - [\mathbf{G}, \mathbf{G}_{r^c}][\mathbf{G}, \mathbf{G}_{r^c}]^+$  is a projection matrix and  $r' = \text{Rank}(\mathbf{P}_{\mathbf{G}, \mathbf{G}_{r^c}}^\perp)$ . Note that since both  $\mathbf{G}, \mathbf{G}_{r^c}$  are semi-unitary matrices, when  $\mathbf{G}_{r^c}^\dagger \mathbf{G} = \mathbf{0}$ , we can also write  $\mathbf{P}_{\mathbf{G}, \mathbf{G}_{r^c}}^\perp = \mathbf{I} - \mathbf{G} \mathbf{G}^\dagger - \mathbf{G}_{r^c} \mathbf{G}_{r^c}^\dagger$  and in this case  $r' = M - r - r^c$ .

- MMSE based receiver: a matrix  $\hat{\mathbf{G}}_r$  is selected after determining  $r$  SINRs for each matrix in  $\mathcal{C}_r$ . Also selected is the best-companion matrix,  $\hat{\mathbf{G}}_{r^c}$ . We let  $\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_r]$ .

$$\{\hat{\mathbf{G}}_r, \hat{\mathbf{G}}_{r^c}\} = \arg \max_{\mathbf{G} \in \mathcal{C}_r, \mathbf{G}_{r^c} \in \mathcal{C}_{r^c}} \left\{ \sum_{j=1}^r \ln (1 + \text{SINR}_{j,r}^{\text{MMSE}}(\mathbf{G}, \mathbf{G}_{r^c})) \right\} \quad (14)$$

where

$$\text{SINR}_{j,r}^{\text{MMSE}}(\mathbf{G}, \mathbf{G}_{r^c}) = \rho' \mathbf{g}_j^\dagger \mathbf{H} \left( \mathbf{I} + \check{\rho} \mathbf{H}^\dagger \mathbf{P}_{\mathbf{G}, \mathbf{G}_{r^c}}^\perp \mathbf{H} + \rho' \mathbf{H}^\dagger \mathbf{G}_{r^c} \mathbf{G}_{r^c}^\dagger \mathbf{H} + \rho' \mathbf{H}^\dagger \sum_{k \neq j} \mathbf{g}_k \mathbf{g}_k^\dagger \mathbf{H} \right)^{-1} \mathbf{H}^\dagger \mathbf{g}_j \quad (15)$$

Note that the alternate rule derived in Section Appendix A can be readily extended to this scenario. In particular, the user first uses (12) to determine the matrix  $\hat{\mathbf{G}}_r$ . Next, it again uses (12) to determine the matrix  $\hat{\mathbf{G}}_{r^c}$ , but where the search is over  $\mathcal{C}_{r^c}$  and  $\tilde{\mathbf{V}}_{(r)}$  is replaced by  $\hat{\mathbf{V}}_{(r^c)}$ , with  $\hat{\mathbf{V}}_{(r^c)}$  denoting the  $M \times r^c$  matrix formed by the  $r^c$  right singular vectors of  $\mathbf{H}^\dagger$  that correspond to the  $r^c$  smallest singular values. Once  $\hat{\mathbf{G}}_r, \hat{\mathbf{G}}_{r^c}$  are determined, the user can compute the  $r$  SINRs corresponding to  $\hat{\mathbf{G}}_r, \hat{\mathbf{G}}_{r^c}$  using (15). Alternatively, the user can compute the best companion PMI using the aforementioned rules. It can then report the CQI(s) that are based on SINR(s) without the best companion (for instance using (5) with  $\hat{\mathbf{G}}_r$ ) along with a delta CQI that corresponds to the average difference between the SINRs without the best companion and those with the best companion ((15) with  $\hat{\mathbf{G}}_r, \hat{\mathbf{G}}_{r^c}$ ).

## Appendix B Derivations of Proposed PMI Selections

We first derive the capacity metric for selecting a PMI of rank  $r$ , where  $1 \leq r \leq N$ . Suppose that the user of interest chooses any arbitrary combining matrix  $\mathbf{A}_r \in \mathbb{C}^{N \times r}$  of rank  $r$  and considers reporting any PMI  $\mathbf{G}_r \in \mathbb{C}^{M \times r}$  to the eNodeB. The user assumes that upon doing so, the transmit precoder employed by the eNodeB to serve it will be  $\mathbf{V}_1 = \mathbf{G}_r$  and that it will be co-scheduled with other users who in turn are served using transmit precoders that lie in the null-space of  $\mathbf{V}_1^\dagger$ , i.e.,  $\mathbf{V}_1^\dagger \mathbf{V}_k = \mathbf{0}$ ,  $\forall k \neq 1$ . In addition it assumes that there will be  $S - r$  such co-scheduled streams (intended for the other users) in total. Thus, the model

that the user deems will be seen by it post-scheduling is

$$\mathbf{y} = \mathbf{A}_r^\dagger \mathbf{H}^\dagger \mathbf{G}_r \mathbf{s}_1 + \sum_{k \neq 1} \mathbf{A}_r^\dagger \mathbf{H}^\dagger \mathbf{V}_k \mathbf{s}_k + \mathbf{A}_r^\dagger \boldsymbol{\eta}, \quad (16)$$

where  $\mathbf{H} \in \mathbb{C}^{M \times N}$  is the channel matrix and  $\boldsymbol{\eta} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$  is the additive noise. In order to determine a rate that can be obtained over this model the user proceeds as follows. It determines an SVD of the combined channel  $\mathbf{A}_r^\dagger \mathbf{H}^\dagger$  as

$$\mathbf{A}_r^\dagger \mathbf{H}^\dagger = \mathbf{U} \boldsymbol{\Lambda} \tilde{\mathbf{Q}}^\dagger, \quad (17)$$

where  $\mathbf{U} \in \mathbb{C}^{r \times r}$  and  $\tilde{\mathbf{Q}} \in \mathbb{C}^{M \times r}$  are unitary and semi-unitary matrices, respectively. Let  $\mathbf{P} = \mathbf{G}_r \mathbf{G}_r^\dagger$  be an orthogonal projection and let  $\mathbf{P}_\perp = \mathbf{I} - \mathbf{P}$  denote its orthogonal complement. The user then obtains the following decomposition of the matrix  $\tilde{\mathbf{Q}}$ , which we recall contains the right singular vectors of the combined channel matrix  $\mathbf{A}_r^\dagger \mathbf{H}^\dagger$ .

$$\tilde{\mathbf{Q}} = \mathbf{P} \tilde{\mathbf{Q}} + \mathbf{P}_\perp \tilde{\mathbf{Q}}. \quad (18)$$

Using (17), (18) along with the assumption that  $\mathbf{G}_r^\dagger \mathbf{V}_k = \mathbf{0}$ ,  $\forall k \neq 1$ , the model in (16) simplifies to

$$\mathbf{y} = \mathbf{U} \boldsymbol{\Lambda} \tilde{\mathbf{Q}}^\dagger \mathbf{P} \mathbf{G}_r \mathbf{s}_1 + \sum_{k \neq 1} \mathbf{U} \boldsymbol{\Lambda} \tilde{\mathbf{Q}}^\dagger \mathbf{P}_\perp \mathbf{V}_k \mathbf{s}_k + \mathbf{A}_r^\dagger \boldsymbol{\eta}. \quad (19)$$

Note that the gain matrix corresponding to the signal component is given by  $\rho' \mathbf{U} \boldsymbol{\Lambda} \tilde{\mathbf{Q}}^\dagger \mathbf{P} \tilde{\mathbf{Q}} \boldsymbol{\Lambda} \mathbf{U}^\dagger$  and the noise covariance matrix is given by  $\mathbf{A}_r^\dagger \mathbf{A}_r$ . Then, in order to determine a rate the user must obtain the covariance matrix of the interference. However, since the transmit precoders employed for the co-scheduled users are not known, the user employs an expected covariance matrix of the interference instead, where the expectation is taken after conditioning on all terms involved in the gain and noise covariance matrices. Thus the rate computed

by the user for the choice of  $\mathbf{A}_r, \mathbf{G}_r$  is given by

$$\ln \left| \mathbf{A}_r^\dagger \mathbf{A}_r + \rho' \mathbf{U} \mathbf{\Lambda} \tilde{\mathbf{Q}}^\dagger \mathbf{P} \tilde{\mathbf{Q}} \mathbf{\Lambda} \mathbf{U}^\dagger + E \left[ \rho' \mathbf{U} \mathbf{\Lambda} \tilde{\mathbf{Q}}^\dagger \mathbf{P}_\perp \left( \sum_{k \neq 1} \mathbf{V}_k \mathbf{V}_k^\dagger \right) \mathbf{P}_\perp \tilde{\mathbf{Q}} \mathbf{\Lambda} \mathbf{U}^\dagger \right] \right| \\ - \ln \left| \mathbf{A}_r^\dagger \mathbf{A}_r + E \left[ \rho' \mathbf{U} \mathbf{\Lambda} \tilde{\mathbf{Q}}^\dagger \mathbf{P}_\perp \left( \sum_{k \neq 1} \mathbf{V}_k \mathbf{V}_k^\dagger \right) \mathbf{P}_\perp \tilde{\mathbf{Q}} \mathbf{\Lambda} \mathbf{U}^\dagger \right] \right| \quad (20)$$

Once (20) is determined the user can further optimize over  $\mathbf{A}_r$  to obtain an optimized estimated rate corresponding to its choice of  $\mathbf{G}_r$ .

To evaluate (20), we need to determine the conditional expectation. This can be accomplished as follows. Consider the SVD of the channel matrix

$$\mathbf{H}^\dagger = \mathbf{R} \mathbf{D} \mathbf{Q}^\dagger, \quad (21)$$

where  $\mathbf{R} \in \mathbb{C}^{N \times N}$  is a unitary matrix and  $\mathbf{Q} \in \mathbb{C}^{M \times N}$  is such that  $\mathbf{Q}^\dagger \mathbf{Q} = \mathbf{I}$ . We assume that  $\mathbf{Q}$  is isotropically distributed and is independent of  $\mathbf{R}, \mathbf{D}$ . Note that this assumption is true when  $\mathbf{H} = \mathbf{H}_w$ , where  $\mathbf{H}_w$  has i.i.d. complex Gaussian elements [11].<sup>1</sup> Then, consider the following SVD

$$\mathbf{A}_r^\dagger \mathbf{R} \mathbf{D} = \mathbf{U} \mathbf{\Lambda} \mathbf{W}^\dagger, \quad (22)$$

where  $\mathbf{U} \in \mathbb{C}^{r \times r}$  and  $\mathbf{W} \in \mathbb{C}^{N \times r}$  are unitary and semi-unitary matrices, respectively. Thus, the SVD of  $\mathbf{A}_r^\dagger \mathbf{H}$  can be written as (17) where  $\tilde{\mathbf{Q}} = \mathbf{Q} \mathbf{W}$ . Note that since  $\mathbf{Q}$  is isotropically distributed and is independent of  $\mathbf{R}, \mathbf{D}, \mathbf{A}_r$ , we have the following result.

**Lemma 1** *The matrix  $\tilde{\mathbf{Q}} = \mathbf{Q} \mathbf{W} \in \mathbb{C}^{M \times r}$  is an isotropically distributed semi-unitary matrix and is independent of  $\mathbf{U}, \mathbf{\Lambda}, \mathbf{A}_r$ .*

Now consider the following decomposition of the matrix  $\tilde{\mathbf{Q}}$ .

$$\tilde{\mathbf{Q}} = \mathbf{P} \tilde{\mathbf{Q}} + \mathbf{P}_\perp \tilde{\mathbf{Q}}. \quad (23)$$

---

<sup>1</sup>We will consider other channel statistics in the sequel.

The following lemma follows from a result in [3].

**Lemma 2** *The matrix  $\mathbf{P}\tilde{\mathbf{Q}}$  can be expanded as  $\mathbf{P}\tilde{\mathbf{Q}} = \mathbf{G}_r\mathbf{X}\mathbf{J}$ , where  $\mathbf{X} \in \mathbb{C}^{r \times r}$  is an isotropically distributed unitary matrix,  $\mathbf{J} \in \mathbb{C}^{r \times r}$  is a lower triangular matrix with positive diagonal elements and where  $\mathbf{G}_r, \mathbf{X}, \mathbf{J}$  are mutually independent. Further, the matrix  $\mathbf{P}_\perp\tilde{\mathbf{Q}}$  can be expanded as  $\mathbf{P}_\perp\tilde{\mathbf{Q}} = \mathbf{F}\mathbf{K}$ , where  $\mathbf{F} \in \mathbb{C}^{M \times r}$  and  $\mathbf{K} \in \mathbb{C}^{r \times r}$  are mutually independent with  $\mathbf{K}^\dagger\mathbf{K} = \mathbf{I} - \mathbf{J}^\dagger\mathbf{J} = \tilde{\mathbf{Q}}^\dagger\mathbf{P}_\perp\tilde{\mathbf{Q}}$ . Moreover,  $\mathbf{F}$  is isotropically distributed in the range of  $\mathbf{P}_\perp$  and  $\mathbf{K} \in \mathbb{C}^{r \times r}$  is a lower triangular matrix with positive diagonal elements that has a one-to-one mapping to  $\mathbf{J}$ .*

Using these results, we have that

$$\begin{aligned}\tilde{\mathbf{Q}}^\dagger\mathbf{G}_r\mathbf{G}_r^\dagger\tilde{\mathbf{Q}} &= \tilde{\mathbf{Q}}^\dagger\mathbf{P}\tilde{\mathbf{Q}} = \mathbf{J}^\dagger\mathbf{X}^\dagger\mathbf{G}_r^\dagger\mathbf{G}_r\mathbf{X}\mathbf{J} = \mathbf{J}^\dagger\mathbf{J}, \\ \tilde{\mathbf{Q}}^\dagger\mathbf{V}_k\mathbf{V}_k^\dagger\tilde{\mathbf{Q}} &= \mathbf{K}^\dagger\mathbf{F}^\dagger\mathbf{V}_k\mathbf{V}_k^\dagger\mathbf{F}\mathbf{K}, \quad \forall k \neq 1.\end{aligned}\tag{24}$$

We can then re-write the expression in (20) as

$$\begin{aligned}\ln \left| \mathbf{A}_r^\dagger\mathbf{A}_r + \rho'\mathbf{U}\mathbf{\Lambda}\mathbf{J}^\dagger\mathbf{J}\mathbf{\Lambda}\mathbf{U}^\dagger + \rho'E \left[ \mathbf{U}\mathbf{\Lambda}\mathbf{K}^\dagger \left( \sum_{k \neq 1} \mathbf{F}^\dagger\mathbf{V}_k\mathbf{V}_k^\dagger\mathbf{F} \right) \mathbf{K}\mathbf{\Lambda}\mathbf{U}^\dagger \right] \right| - \\ \ln \left| \mathbf{A}_r^\dagger\mathbf{A}_r + \rho'E \left[ \mathbf{U}\mathbf{\Lambda}\mathbf{K}^\dagger \left( \sum_{k \neq 1} \mathbf{F}^\dagger\mathbf{V}_k\mathbf{V}_k^\dagger\mathbf{F} \right) \mathbf{K}\mathbf{\Lambda}\mathbf{U}^\dagger \right] \right|\end{aligned}\tag{25}$$

Next, the expected value in (25) is computed after keeping the other terms  $\mathbf{U}, \mathbf{\Lambda}, \mathbf{A}_r, \mathbf{J}$  (which determine the gain and noise matrices) fixed. Note that fixing  $\mathbf{J}$  also fixes  $\mathbf{K}$ . Thus the (conditional) expected value in (20) is computed by replacing the term  $\sum_{k \neq 1} \mathbf{F}^\dagger\mathbf{V}_k\mathbf{V}_k^\dagger\mathbf{F}$  by its (conditional) expected value. However, since  $\mathbf{F}, \{\mathbf{V}_k\}_{k \neq 1}$  are independent of  $\mathbf{U}, \mathbf{\Lambda}, \mathbf{A}_r, \mathbf{J}$ , their distributions are not impacted by such conditioning. In addition  $\mathbf{F}, \{\mathbf{V}_k\}_{k \neq 1}$  are isotropically distributed in the  $M - r$  dimensional sub-space spanned by  $\mathbf{P}_\perp$ . Then, it can be concluded that each matrix  $\mathbf{F}^\dagger\mathbf{V}_k\mathbf{V}_k^\dagger\mathbf{F}$  has a matrix variate beta-distribution and

that

$$E\left[\sum_{k \neq 1} \mathbf{F}^\dagger \mathbf{V}_k \mathbf{V}_k^\dagger \mathbf{F}\right] = \frac{S-r}{M-r} \mathbf{I}. \quad (26)$$

Using (26) in (25) an approximate rate is determined as

$$\ln \left| \mathbf{A}_r^\dagger \mathbf{A}_r + \rho' \mathbf{A}_r^\dagger \mathbf{H}^\dagger \mathbf{P} \mathbf{H} \mathbf{A}_r + \rho' \frac{S-r}{M-r} \mathbf{A}_r^\dagger \mathbf{H}^\dagger \mathbf{P}_\perp \mathbf{H} \mathbf{A}_r \right| - \ln \left| \mathbf{A}_r^\dagger \mathbf{A}_r + \rho' \frac{S-r}{M-r} \mathbf{A}_r^\dagger \mathbf{H}^\dagger \mathbf{P}_\perp \mathbf{H} \mathbf{A}_r \right| \quad (27)$$

Maximizing (27) over  $\mathbf{A}_r$ , after some manipulations, the optimized rate is given by

$$\ln \left| \mathbf{I} + \rho' \mathbf{H}^\dagger \mathbf{P} \mathbf{H} + \rho' \frac{S-r}{M-r} \mathbf{H}^\dagger \mathbf{P}_\perp \mathbf{H} \right| - \ln \left| \mathbf{I} + \rho' \frac{S-r}{M-r} \mathbf{H}^\dagger \mathbf{P}_\perp \mathbf{H} \right| \quad (28)$$

The user thus employs the rule in (3) which optimizes (28) over all  $\mathbf{G}_r$  in the codebook of rank- $r$  semi-unitary matrices. The other metrics given in Appendix A can be obtained similarly.

Finally, some comments are in order regarding extensions to other channel statistics. It can be noted that the techniques used above readily accommodate a transformed channel matrix  $\tilde{\mathbf{H}}^\dagger = \mathbf{C} \mathbf{H}^\dagger$ , where  $\mathbf{C} \in \mathbb{C}^{N \times N}$  is any arbitrary non-singular matrix independent of  $\mathbf{H}$ . Thus, any receive correlation and inter-cell interference whitening by the UE can be accounted for. While, a formal justification of the formula for the important case of transmit correlation has not been provided, we believe it is quite applicable in such scenarios as well, the special case of rank-1 PMI was found to work well over correlated scenarios in [5]. Note that a nice feature of (28) is that as the reported PMI  $\mathbf{G}_r$  more finely captures all dominant right singular vectors of  $\mathbf{H}^\dagger$ , the contribution of the interference term goes to zero.

## References

- [1] R1-103346, "Simulation assumptions Rel-10 feedback for 4Tx," Montreal, May 2010

- [2] NEC Group, "Results for MU-MIMO under 4 TX Evaluation Assumptions," *3GPP TSG RAN WG1 R1-103831* 61bis, Dresden, Germany, June. 2010.
- [3] N. Ravindran, and N. Jindal, "Limited Feedback-Based Block Diagonalization for the MIMO Broadcast Channel," *IEEE J. Sel. Areas. Commun.*, Vol. 26, No. 8, pp. 1473-1482, Oct. 2008.
- [4] M. Trivellato, F. Boccardi, and H. Huang, "On Transceiver Design and Channel Quantization for Downlink Multiuser MIMO Systems with Limited Feedback," *IEEE J. Sel. Areas. Commun.*, Vol. 26, no. 8, pp. 1494-1504, Oct. 2008.
- [5] NTT DoCoMo, "Investigation on Enhanced DL MU-MIMO Processing Based on Channel Vector Quantization for LTE-Advanced," *3GPP TSG RAN WG1 R1-094242* 58bis, Miyazaki, Japan, Oct. 2009.
- [6] LGE, "Investigation on Feedback for MU-MIMO," *3GPP TSG RAN WG1 R1-101239* 60, San-Francisco, USA, Feb. 2010.
- [7] TI, "Multi-rank Implicit Feedback for MU-MIMO," *3GPP TSG RAN WG1 R1-101089* 60, San-Francisco, USA, Feb. 2010.
- [8] Samsung, "Generalized transformation for adaptive codebooks," *3GPP TSG RAN WG1 R1-101163* 60, San-Francisco, USA, Feb. 2010.
- [9] ALU, ALU Shanghai Bell, "Best companion reporting for single-cell MU-MIMO pairing," *3GPP TSG RAN WG1 R1-094613*.
- [10] NEC Group, "MU-MIMO: CQI Computation and PMI Selection," *3GPP TSG RAN WG1 R1-103832*.
- [11] R. J. Muirhead, "Aspects of Multivariate Statistical Theory," *Wiley 1982*.
- [12] Huawei et. al. "Simulation assumptions Rel-10 feedback for 4Tx," *3GPP TSG RAN WG1 R1-103346*, Montreal, May 2010

<b>Parameter</b>	<b>Assumptions used for evaluation</b>
Deployment scenario	IMT Urban Micro (UMi)
Duplex method and bandwidths	FDD: 10MHz for downlink
Cell layout	Hexagonal grid 19 sites, 3 cells per site
Number of UEs per sector	10
Network synchronization	Synchronized
Antenna configuration (eNB)	4 TX co-polarized antennas with 0.5-lambda spacing
Antenna configuration (UE)	2 RX co-polarized antennas with 0.5-lambda spacing
Downlink transmission scheme	MU-MIMO: Maximum 2 co-scheduled UEs per RB. Each UE can have rank-1 or rank-2
Codebook	Rel.8 codebook
Downlink scheduler	1. Max Rate 2. Proportional fair in time and frequency Scheduling granularity: 1 RB or 5 RBs
Feedback assumptions	Report is with 5ms periodicity and 4ms delay. Sub-band CQI and PMI without measurement or feedback errors. Sub-band granularity: 1 or 5 RBs
Downlink HARQ scheme	Chase Combining
Downlink receiver type	LMMSE.
Channel estimation error	NA
Feedback channel error	NA
Control channel and reference signal overhead	3 OFDM symbols for control. Use TBS tables in 3GPP TS 36.213 for throughput calculation.

Table 1: Simulation assumptions.

<b>SU-MIMO 4x2</b>	Average Cell Spectral Efficiency (bits/s/Hz)	5% Cell Edge Spectral Efficiency (bits/s/Hz)
Rel.8 sub-band PMI feedback + sub-band CQI	2.2583	0.0736

Table 2: Spectral efficiency performance of SU-MIMO (4x2) with proportional fair scheduling (sub-band size 1RB).

<b>SU-MIMO 4x2</b>	Average Cell Spectral Efficiency (bits/s/Hz)	5% Cell Edge Spectral Efficiency (bits/s/Hz)
Rel.8 sub-band PMI feedback + sub-band CQI	2.1488	0.0679

Table 3: Spectral efficiency performance of SU-MIMO (4x2) with proportional fair scheduling (sub-band size 5RB).

<b>MU-MIMO 4x2</b> <b>With MU Report</b> (Rel.8 sub-band PMI feedback + sub-band CQI)	Average Cell Spectral Efficiency (bits/s/Hz)
UE assumes S=2 scheduled streams	2.6582
UE assumes S=3 scheduled streams	3.0452
UE assumes S=4 scheduled streams	2.9057

Table 4: Spectral efficiency performance of MU-MIMO (4x2) with proposed MU report and various values of S with maximum sum-rate scheduling (Sub-band size 1RB, near orthogonal transmit precoding without ZF).

<b>MU-MIMO 4x2</b> <b>With MU Report (S=3)</b>	Average Cell Spectral Efficiency (bits/s/Hz)	5% Cell Edge Spectral Efficiency (bits/s/Hz)
Rel.8 sub-band PMI feedback + sub-band CQI	2.4556	0.0874

Table 5: Spectral efficiency performance of MU-MIMO (4x2) with proposed MU report (S=3) using proportional fair scheduling (sub-band size 1RB, near-orthogonal transmit precoding without ZF).

<b>MU-MIMO 4x2</b> <b>With MU Report (S=3)</b>	Average Cell Spectral Efficiency (bits/s/Hz)	5% Cell Edge Spectral Efficiency (bits/s/Hz)
Rel.8 sub-band PMI feedback + sub-band CQI	2.3321	0.0734

Table 6: Spectral efficiency performance of MU-MIMO (4x2) with proposed MU report (S=3) using proportional fair scheduling (near-orthogonal transmit precoding with ZF and subband size 5RB).