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1. Introduction

LTE-Advanced (LTE-A) targets an instantaneous downlink peak data rate of 1 Gbps [3]. In addition to increasing the total bandwidth, one way to achieve this target is by increasing the number of spatially-multiplexed data streams towards a single-user (SU). Proposals to employ up to 8 antennas at both eNodeB and UE have been made, considering in particular up to 4 antennas on a handset and 8 antennas on a laptop.
The theoretical capacity of an *uncorrelated* SU-MIMO channel is well known since the pioneering works of Telatar [1]. However, the antennas in practical devices cannot be spaced arbitrarily. Due to the physical dimensions the antennas tend to be correlated.
For a given surface area of a transmitting device and a receiving device, it is difficult to say how many antennas can be employed before the mutual correlation cancels their benefits. Of course the number of antennas also depends on power consumption and other design aspects. In the following we will focus only on antenna correlation to determine an upper bound on the reasonable number of antennas that is likely to be useful. Commonly, it is acknowledged that any pair of antennas should be spaced at least by $\lambda/2$, but no strong theoretical argument supports this claim. Some recent papers [4],[5] shed some light on the spatial limitations of MIMO. In this document we:

- analyze the capacity gains given by $N \times N$ MIMO systems on devices with limited surface areas
- show that, even for a mobile phone terminal, the 8×8 MIMO setup can provide significant capacity gain
- show that, for large systems such as laptops, even 16×16 MIMO is theoretically interesting
- discuss the different problems practically encountered to reach such transmission rates.

The remainder of this contribution is organized as follows: Section 2 introduces and justifies the Kronecker channel model. Section 3 provides the formula for the capacity limit of *dense* MIMO setups which approximates the SU-MIMO capacity. Furthermore it provides a practical algorithm to compute the theoretical limit for a given transceiver design. Section 4 gives simulation results for various transceiver designs corroborating the theoretical claims. Section 5 discusses several key practical points to achieve high transmission rates. Finally, Section 6 states our conclusions.

2. System model, justification and usage

We consider a SU-MIMO downlink system with N_{T} transmit antennas at the eNodeB and N_{R} receive antennas at the UE. Both eNodeB and UE have a limited surface, which as a consequence entails correlation at the antenna arrays. The prior information on the communication channel can be summarized as: (i) the channel has a known mean power gain, (ii) the distances $d_{i,j}^{(\mathrm{T})}$ and $d_{i,j}^{(\mathrm{R})}$ between the transmit antennas (i,j) and the receive antennas (i,j) respectively are known, respectively. This limited amount of knowledge implies that the Kronecker channel model is the most appropriate model given the prior information on the channel. This is demonstrated in [2]. Therefore, using $\mathbf{\Theta}_{\mathrm{T}}$ to denote the correlation matrix at the transmitter, $\mathbf{\Theta}_{\mathrm{T}}$ the correlation matrix at the receiver and $\mathbf{H}_w$ some standard Gaussian i.i.d. distributed $N_{\mathrm{R}} \times N_{\mathrm{T}}$ complex matrix, the channel model $\mathbf{H}$ reads

$$\mathbf{H} = \mathbf{\Theta}_{\mathrm{R}}^{1/2} \mathbf{H}_w \mathbf{\Theta}_{\mathrm{T}}^{1/2} \tag{1}$$

Given the array of distances $\{d_{i,j}^{(\mathrm{R})}\}$ at the receiver for instance, we show, using the classical Jake's model[1] that $\mathbf{\Theta}_{\mathrm{R}}$ is of the form

$$\begin{bmatrix} 1 & J_0(\frac{2\pi}{\lambda} d_{1,2}^{(\mathrm{R})}) & J_0(\frac{2\pi}{\lambda} d_{1,3}^{(\mathrm{R})}) & \dots & J_0(\frac{2\pi}{\lambda} d_{1,N_{\mathrm{R}}}^{(\mathrm{R})}) \\ J_0(\frac{2\pi}{\lambda} d_{2,1}^{(\mathrm{R})}) & 1 & J_0(\frac{2\pi}{\lambda} d_{2,3}^{(\mathrm{R})}) & \dots & J_0(\frac{2\pi}{\lambda} d_{2,N_{\mathrm{R}}}^{(\mathrm{R})}) \\ J_0(\frac{2\pi}{\lambda} d_{3,1}^{(\mathrm{R})}) & J_0(\frac{2\pi}{\lambda} d_{3,2}^{(\mathrm{R})}) & 1 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ J_0(\frac{2\pi}{\lambda} d_{N_{\mathrm{R}},1}^{(\mathrm{R})}) & J_0(\frac{2\pi}{\lambda} d_{N_{\mathrm{R}},2}^{(\mathrm{R})}) & \dots & \dots & 1 \end{bmatrix} \tag{2}$$

In the following we consider the impact of correlation on the resulting achievable transmission rate. In order to establish an upper bound for the capacity, we especially consider the asymptotic case for which the number of antennas grows to infinity at both transmitter and receiver with a constant ratio $N_{\mathrm{T}}/N_{\mathrm{R}} = \beta$: this is referred to as the *dense* MIMO model or *dense* antenna model. This asymptotic study is interesting in the sense that the capacity achieved for an infinite number of antennas is already achieved for the "best" $N \times N$ scenario with N generally fairly small.

3. Physical limits of a MIMO system

We show in this section that the capacity achieved by the $N \times N$ MIMO setups saturates quickly when N grows to a quasi-optimal scenario. By quasi-optimal, we mean that adding more antennas does not contribute to a significant increase in capacity. We also show that the peak rate corresponding to this quasi-optimal scenario can always be computed; for this, one needs to consider the asymptotic dense MIMO capacity which was recently shown to be easy to compute. A simple algorithm allows us to assess exactly this capacity limit, which only depends on the shape and surface area of both transmitter and receiver.

[1] which is the least committal model given the prior assumptions on the antenna correlations

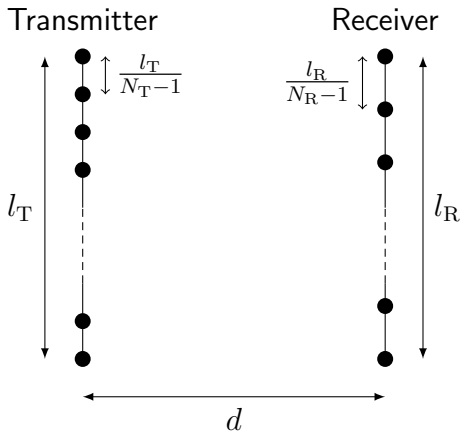

Figure 1: One-dimensional antenna array geometry

In a recent contribution [5] it is shown that the asymptotic (dense antenna) capacity of a correlated system only depends on the ratio "wavelength" over "system dimensions". For instance, consider first a setup of one-dimensional arrays as depicted in Figure 1. The transmitter is a line of length l_{T} while the receiver is a line of size l_{R}. In this case the entries of the correlation matrices are simple. For instance, we have

$$[\mathbf{\Theta}_{\mathrm{R}}]_{i,j} = J_0\left(\frac{2\pi}{\lambda} d_{i,j}^{\mathrm{R}}\right) = J_0\left(\frac{2\pi}{\lambda}\frac{|i-j|l_{\mathrm{R}}}{N_{\mathrm{R}}-1}\right) \tag{3}$$

Call $F_{\mathrm{R}}(k)$ the kth eigenvalue of $\mathbf{\Theta}_{\mathrm{R}}$, $k \in \{1, \ldots, N_{\mathrm{R}}\}$ and $F_{\mathrm{T}}(k)$ the kth eigenvalue of $\mathbf{\Theta}_{\mathrm{T}}$. We consider an interference-limited scenario, such that the dominant interference is correlated at all the receive antennas. The total receive SNR, denoted ρ', is therefore invariant with the number of antennas, which are considered to be dipoles perpendicular to the surface area of the receiver in question.

The general formulation for the ergodic capacity $\mathcal{C}$ of the asymptotic dense MIMO setup is shown to be a function of the parameters ρ', $F_{\mathrm{R}}(k)$ $(1 \leq k \leq N_{\mathrm{R}})$ and $F_{\mathrm{T}}(k)$ $(1 \leq k \leq N_{\mathrm{T}})$ [5]. The formula is provided in the appendix.

This result allows us to find the dense MIMO capacity via a fixed point algorithm on the variables Γ and Υ, which we also describe in the appendix.

Simulations indicate that the theoretical capacity limit is slightly lower compared to the optimal capacity, when the antennas are uniformly distributed on the available surface; this is due to minor edge effects. However, the approximation is fairly accurate and allows us to easily compute the capacity saturation for large MIMO setups without Monte-Carlo simulations over many channel realizations.

Note that the limit capacity only depends on the eigenvalue distribution of the correlation matrices $\mathbf{\Theta}_{\mathrm{R}}$ and $\mathbf{\Theta}_{\mathrm{T}}$. Therefore, we can intuitively understand that, by densifying the antennas, most distances $d_{i,j}^{(\mathrm{R})}$ and $d_{i,j}^{(\mathrm{T})}$ tend to be equal, and by continuity of the Bessel J_0 function, most entries of $\mathbf{\Theta}_{\mathrm{R}}$ and $\mathbf{\Theta}_{\mathrm{T}}$ tend to be equal. Therefore, the effective ranks of those matrices tend to saturate. This means that there should exist a limit scenario $N_{\mathrm{sat}} \times N_{\mathrm{sat}}$ such that, for any

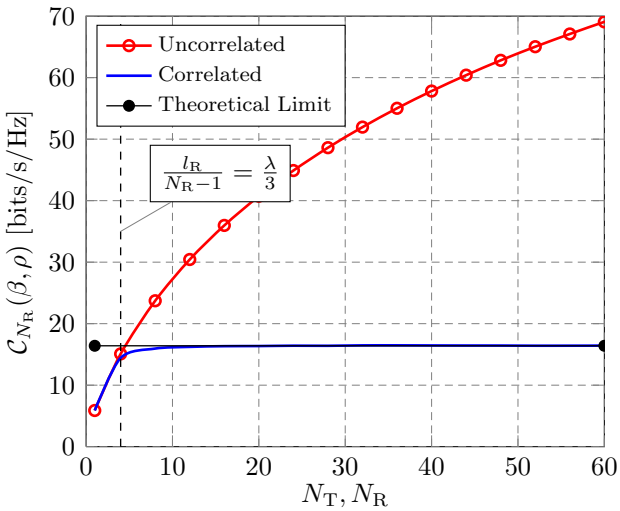

Figure 2: **One-dimensional setup**, ergodic MIMO capacity $\mathcal{C}_{N_R}(\beta,\rho)$, $N_R = N_T$, $\rho' = 20$ dB

denser MIMO setup, the capacity does not increase. As a consequence, the aforementioned asymptotic capacity limit corresponds to the capacity achieved by this $N_{sat} \times N_{sat}$ setup. This is proved by simulations in the following.

4. Simulation and results

In the following, we consider a one-dimensional setup and a two-dimensional setup.

4.1. One-dimensional setup

Consider a line of antennas of length $l_R = 10$ cm at the UE and length $l_T = 3$ m at the eNodeB. The signal wavelength is $\lambda = 10$ cm corresponding to a carrier frequency of 3 GHz. The antennas are uniformly distributed on the linear arrays. The results are depicted in Figure 2. As expected we observe a saturation of the capacity when increasing the number of transmit and receive antennas. The saturation level is shown to be well described by the theoretical limit of the dense MIMO setup. Notice also that, as previously stated, the 4×4 MIMO scenario almost corresponds to the saturated capacity. This means that no more than 4 antennas would be needed in this one-dimensional scenario; this also shows that the asymptotic capacity, which can be easily assessed, is a fairly good approximation of this "best" 4×4 setup. Note also that in this case $l/\lambda = 1$ is non negligible and proves to be detrimental to the capacity. Hence, the achieved capacity is upper-bounded because of the size of the UE.
Indeed, with $N_R = 4$, the capacity scales linearly with $\min(N_R, N_T)$ as in the case of uncorrelated antennas. This scenario corresponds to a minimal distance of $\lambda/3$ between the antennas at the UE.

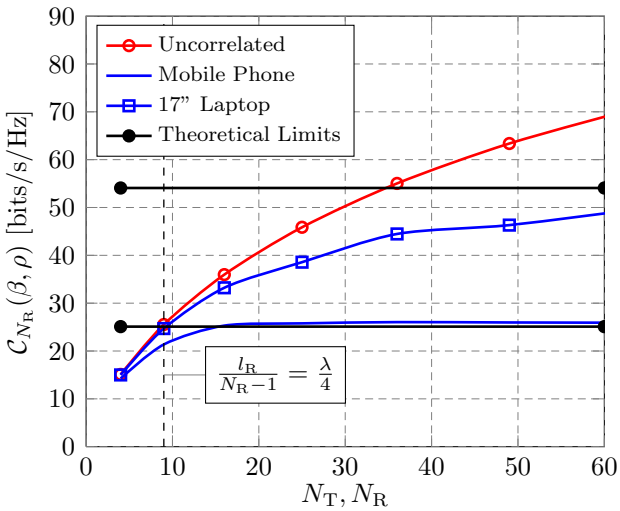

Figure 3: **Two-dimensional setup**, ergodic MIMO capacity $\mathcal{C}_{N_R}(\beta,\rho)$, $N_R = N_T$, $\rho' = 20$ dB

4.2. Two-dimensional setup

Consider now a square surface at the eNodeB of size $1\,\mathrm{m} \times 1\,\mathrm{m}$ with increasing number of antennas and a rectangular UE with dimensions $10\,\mathrm{cm} \times 5\,\mathrm{cm}$ (mobile phone) or $40\,\mathrm{cm} \times 30\,\mathrm{cm}$ (17" laptop). All antennas are uniformly distributed on a grid. The performance in terms of capacity is depicted in Figure 3. The surprising result here is that the capacity scales like the ideal uncorrelated case up to $N_R \simeq 9$ for the mobile phone. This corresponds to the case of a 3×3 grid of antennas, spaced by a minimal distance of $\lambda/4$. Note also that the laptop UE capacity can grow very large and almost fits the uncorrelated scenario as long as $N_R \leq 16$. This suggests that even higher MIMO configurations than 8×8 are theoretically possible.

5. Discussion

As presented previously, and thoroughly detailed in [5], we observe in simulation that the dense MIMO limit is a good approximation of the optimal MIMO capacity for a uniform distribution of the antennas.

We verified that the 4×4 and even 8×8 antenna configurations still achieve a non-negligible capacity growth for both a laptop and a mobile phone. Indeed, in case of a laptop the capacity grows linearly up to a 16×16 MIMO configuration.

It is interesting to compare the achievable capacity for the mobile phone and laptop. With 4 antennas, there is no difference in capacity between a mobile phone and a laptop. Doubling the number of antennas to 8 only enables the laptop to achieve a 20% increase in capacity compared to the mobile phone, while 16 antennas gives a 30% difference.

However, it is important to consider the limitations of these results carefully. The system

model assumes a rich scattering environment[2]. In channels with higher correlation this model is not applicable. The presented results hold only for rich scattering scenarios. Additionally, the model assumes that antennas can be freely located anywhere on the considered surface area. In practice this is an unrealistic assumption, especially in the case of a mobile phone, where the dimension of each antenna in the direction perpendicular to the surface area is strictly limited by the form factor of the phone. Moreover, the presence of all the other components of a mobile phone, or even a laptop, prevent unconstrained positioning of the antennas. Other factors such as cost and weight must also be taken into account.
On the other hand, note also that we did not evaluate the performance of different antenna geometries such as cross-polarized antennas at either the transmitter or the receiver. Such geometries can provide some scope for additional antennas to be fitted into the designs studied above if a proper correlation model for cross-polarized antennas is added.

6. Conclusion

This contribution shows the effect of the limited dimensions of a UE on the theoretical capacity limits for SU-MIMO as the number of antennas is increased. According to the idealised model used here, we observe that, for most reasonable numbers of antennas, a mobile phone can achieve a throughput close to that of a much larger laptop.

[2]expressed by the inner Gaussian matrix $\mathbf{H}_w$

A. Dense MIMO capacity

The capacity of the dense $N_\mathrm{R} \times N_\mathrm{T}$ MIMO system is given by

$$\begin{aligned}\mathcal{C}(\beta, \rho') = &\sum_{k=0}^{N_\mathrm{T}} \log(1 + \beta\rho' F_\mathrm{T}(k)\Gamma) \\ &+ \sum_{k=0}^{N_\mathrm{R}} \log(1 + \rho' F_\mathrm{R}(k)\Upsilon) \\ &- \beta \cdot \rho' \cdot \Gamma(\rho')\Upsilon(\rho') \log(e)\end{aligned} \tag{4}$$

with

$$\Gamma(\rho') = \frac{1}{\beta N_\mathrm{R}} \sum_{k=0}^{N_\mathrm{R}} \frac{N_\mathrm{R} F_\mathrm{R}(k)}{1 + \rho' F(k)\Upsilon} \tag{5}$$

$$\Upsilon(\rho') = \frac{1}{N_\mathrm{T}} \sum_{k=0}^{N_\mathrm{T}} \frac{N_\mathrm{T} F_\mathrm{T}(k)}{1 + \rho'\beta F(k)\Gamma}. \tag{6}$$

B. Algorithm

The necessary steps to compute the previously defined capacity dense MIMO $\mathcal{C}$ can be summarized as follows:

1. For a given system, design the distance relations $d^{(\mathrm{R})}$ and $d^{(\mathrm{T})}$ that must scale with N_R and N_T.

2. Generate the corresponding correlation matrices $\mathbf{\Theta}_\mathrm{R}$ and $\mathbf{\Theta}_\mathrm{T}$.

3. For large[3] N_R and N_T and $N_\mathrm{T}/N_\mathrm{R} = \beta$, compute the eigenvalues $F_\mathrm{R}(k)$, $F_\mathrm{T}(k)$ of $\mathbf{\Theta}_\mathrm{R}$, $\mathbf{\Theta}_\mathrm{T}$, respectively.

4. *Initialization*: Set $\Gamma = 0$, $\Upsilon = 0$.

5. *Loop*: Iteratively compute Γ and Υ according to equations (5) and (6).

6. *Termination*: After a sufficient number of iterations, compute the capacity $\mathcal{C}$ according to formula (4).

[3] e.g. of order 256 to 1024

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[4] T.S. Pollock, T.D. Abhayapala, and R.A. Kennedy, “Antenna saturation effects on dense array MIMO capacity,” in IEEE Intl. Conf. on Commun., ICC’03, Anchorage, Alaska, May. 11-15 2003, pp. 2301-2305.

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