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Title:	Unitary Beamforming for MU-MIMO With Per Antenna Power Constraint for LTE-A
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1. Introduction

Multiple-Input Multiple-Output (MIMO) systems have the potential to significantly increase the throughput [1]. The sum-rate achieved on a MU-MIMO downlink is highly dependent on the precoding scheme and the channel state information available at the transmitter (CSIT). The capacity of the MIMO-BC channel is achieved by encoding the transmit symbols according to the dirty-paper strategy. Linear precoding techniques such as zero-forcing beamforming (ZFBF), regularized ZFBF (R-ZFBF) or unitary beamforming (UBF) can achieve a large portion of the MIMO-BC capacity, as already shown in [2]. UBF has the advantage that the signal-to-interference and noise ratio (SINR) at the receiver can be computed exactly and that it is robust to channel estimation errors. Of particular interest is the case when the UBF is further constrained to have constant modulus elements i.e. all elements have the same magnitude. The reason is that this structure does not increase the peak-to-average power ratio (PAPR) of the signal prior to the amplification. The PAPR is important for the design and efficiency of the transmit power amplifiers. A high PAPR requires a high dynamic range over a large bandwidth which leads to amplifiers that are expansive and power inefficient and hence leads to base-stations with high power consumption. Note that already the OFDM scheme leads to increased PAPR and therefore a further increase caused by the precoding is undesirable. This paper compares the various precoding schemes in terms of achievable sum-rate. We draw conclusions on the trade-off between sum-rate and the several constraints on the beamforming matrix.

2. System Model

We assume the MU-MIMO downlink scenario where an eNodeB with M antennas communicates to K single-antenna UEs and $K \geq M$. The K users are separated by their spatial signature or beamforming vector $\mathbf{v}_k$. The transmit signal is formed as $\mathbf{x} = \sqrt{\frac{P}{M}} \sum_{k=1}^{K} \mathbf{v}_k s_k$. The information symbols s_k per user have unit power i.e. $|s_k|^2 = 1$. For an OFDM-based system the input/output equation reads

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where the channel matrix $\mathbf{H} = [\mathbf{h}_1 \mathbf{h}_2 \dots \mathbf{h}_K]^{\mathsf{H}} \in \mathbb{C}^{K \times M}$ has independent and circularly symmetric standard Gaussian entries. The noise vector is Gaussian distributed with $\mathbf{n} \sim \mathcal{N}_c(\mathbf{0}, \sigma_n^2 \mathbf{I})$. In particular the received signal per user is given by

$$y_k = \sqrt{\frac{P}{M}} \mathbf{h}_k^{\mathsf{H}} \mathbf{v}_k s_k + \sqrt{\frac{P}{M}} \sum_{\substack{j=1 \\ j \neq k}}^{M} \mathbf{h}_k^{\mathsf{H}} \mathbf{v}_j s_j + n_k \tag{2}$$

where $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$ $(k = 1, 2, \dots, K)$ models the channel from the eNodeB to user k. The first term in (2) is the useful signal of user k. The second term is the inter-user interference resulting from the residual correlation between the user's spatial signature $\mathbf{v}_k$ and channel $\mathbf{h}_k$. The last term is the noise which is independent from all other terms. As a result the SINR for user k is given by

$$\gamma_k = \frac{|\mathbf{h}_k^{\mathsf{H}} \mathbf{v}_k|^2}{\sum_{j=1, j \neq k}^{M} |\mathbf{h}_k^{\mathsf{H}} \mathbf{v}_j|^2 + \frac{M \sigma_n^2}{P}} \tag{3}$$

We measure the performance of the different precoding schemes in terms of ergodic sum-rate i.e. the long-term average of the instantaneous sum-rate over all channel realizations.

$$\mathcal{R} = \sum_{k=1}^{M} \log_2 (1 + \gamma_k) \tag{4}$$

3. Linear Beamforming Techniques

This section introduces common linear beamforming techniques where the focus lies on unitary beamforming (UBF). Zero-forcing BF (ZFBF) and Regularized ZFBF (R-ZFBF) have already been presented in [2] and are given in the appendix.

3.1. Unitary beamforming

Unitary beamforming is the current assumption in LTE for MU-MIMO. In general UBF has the advantage that the SINR per user can be computed exactly. As the spatial signatures of the users are orthogonal i.e. $\mathbf{V}_u \mathbf{V}_u^{\mathsf{H}} = \mathbf{I}$, (3) simplifies to

$$\gamma_k = \frac{\|\mathbf{h}_k\|^2 \rho_k^2}{\|\mathbf{h}_k\|^2 (1 - \rho_k^2) + \frac{M \sigma_n^2}{P}} \tag{5}$$

with $\rho_k^2 = |\bar{\mathbf{h}}_k^{\mathsf{H}} \mathbf{v}_k|^2$, $\bar{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$. Hence the SINR of user k is *independent* of the interfering channels as they are transformed into the null-space of $\mathbf{h}_k$. In general the parametrization of $\mathbf{V}_u$ requires the optimization of $M(M-1)$ real parameters[1] ($M-1$ angles and $(M-1)^2$ magnitudes). The optimization with respect to (4) is a non-convex problem and no closed-form solution exists. In [3] an iterative optimization method based on successive Givens rotations was presented. However, it has not been proved that this optimization always converges to the optimal solution as there are discrete degrees of freedom that are ignored by the Givens rotation matrix.

[1] Continues degrees of freedom. Note that one angle is fixed and serves as a reference.

3.2. Constrained unitary beamforming

In this section we analyze the parametrization of a unitary matrix with constant modulus entries i.e. $v_{i,j} = 1 \; \forall i, j$. Imposing this constraint leads to $M - 1$ continues degrees of freedom corresponding to the angles per transmit antenna. An additional advantage of the constrained UBF (CUBF) is that it does not increase the PAPR i.e. there are no stricter requirements on the power-amplifiers. CUBF also proves to be more robust to channel estimation errors as shown in [2].
A possible construction for the CUBF is presented in [4]

$$\mathbf{V} = \mathbf{\Phi}\mathbf{U} \tag{6}$$

with phase matrix $\mathbf{\Phi} = \mathrm{diag}(1 \; e^{j\varphi_2} \; e^{j\varphi_3} \; \dots \; e^{j\varphi_M})^{M\times M}$. The matrix $\mathbf{U} \in \mathbb{C}^{M\times K}$ is a basis unitary matrix with constant modulus entries. An example is given by the Walsh-Hadamard matrix (if $M = 2^i \; (i = 1, 2, \dots)$)

$$\mathbf{U}_i = \begin{pmatrix} \mathbf{U}_{i-1} & \mathbf{U}_{i-1} \\ \mathbf{U}_{i-1} & -\mathbf{U}_{i-1} \end{pmatrix}, \; \mathbf{U}_0 = 1 \tag{7}$$

Alternatively we can use the DFT-matrix

$$\mathbf{U}(m, n) = e^{-j\frac{2\pi}{M}(m-1)(n-1)} \; , \; m, n = 1, 2, \dots, M \tag{8}$$

In general, the optimization of the matrix $U(m, n)$ is possible by adding one additional parameter as explained in [4]. However, in this paper, for the sake of simplicity, we consider only CUBF based on Walsh-Hadamard matrix given in (7), because it is shown to outperform the DFT-based construction for $M = 2^i \; (i = 1, 2, \dots)$.
A closed-form solution of the problem which optimizes the parameters of the CUBF schemes with respect to sum-rate does not exist so far. However, an iterative algorithm can be found in [4] and it is used here to obtain the simulation results.

3.3. Codebook-based unitary beamforming

The UBF is constrained to be an element of a predefined codebook. The codebook size is limited by the amount of signalling available to indicate vectors in the codebook. Here we consider the codebook defined in LTE Rel-8 for SU-MIMO [5] which can be seen as one possible quantisation of the CUBF. The eNodeB chooses the precoding matrix that maximizes the sum-rate among all the matrices defined in the codebook. The advantages of using a shared codebook are: reduced feedback overhead and reduced complexity in user selection process. However, the performance are limited compared to optimized UBF and CUBF as shown in the next section.

4. Simulation and Results

In this section we compare the performance in terms of ergodic sum-rate of the various precoding schemes. Figures 1 and 2 present the performance for a 2×2 and 4×4 MIMO

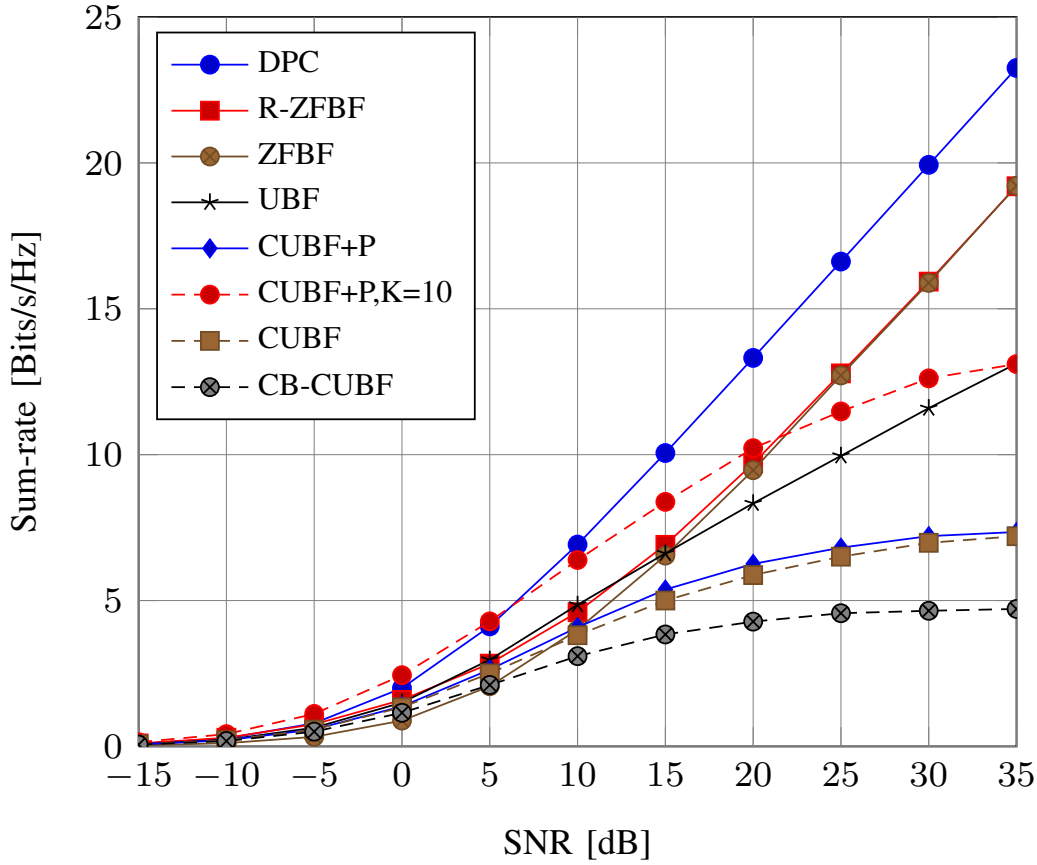

Figure 1: 2×2 MIMO independent Rayleigh fading channel, 1e4 channel realizations

configuration, respectively. In Figure 1 the curve labeled "$+P$" show the results of CUBF with additional power optimization and "$K = 10$" shows the performance of CUBF when a larger ($K = 10$) pool of users is available and the user diversity can be exploited. This is the topic discussed in [6].
We observe in both figures that UBF techniques outperform the ZFBF at low/medium SNR, e.g. in the 4×4 MIMO case at SNR of 10 dB, UBF gains more than 3 Bits/s/Hz. CUBF achieves better performance than CB-UBF (sum-rate gain of $40 - 50\%$). However it can be noticed that CUBF and CBUBF suffer from a drastic loss in performance compared to UBF. In fact, by imposing the constant modulus constraint we drastically reduce the number of degrees of freedom in the beamforming matrix. Consequently, CUBF cannot adapt to the increased channel dimension and the performance decreases.
Moreover note that R-ZFBF, ZFBF and UBF scale linearly with the number of transmit and receive antennas whereas this is not the case for the constrained UBF.

We clearly see that practical restrictions on the PAPR of the transmit signal before the amplification dramatically reduce the achievable sum-rate on the MU-MIMO downlink.

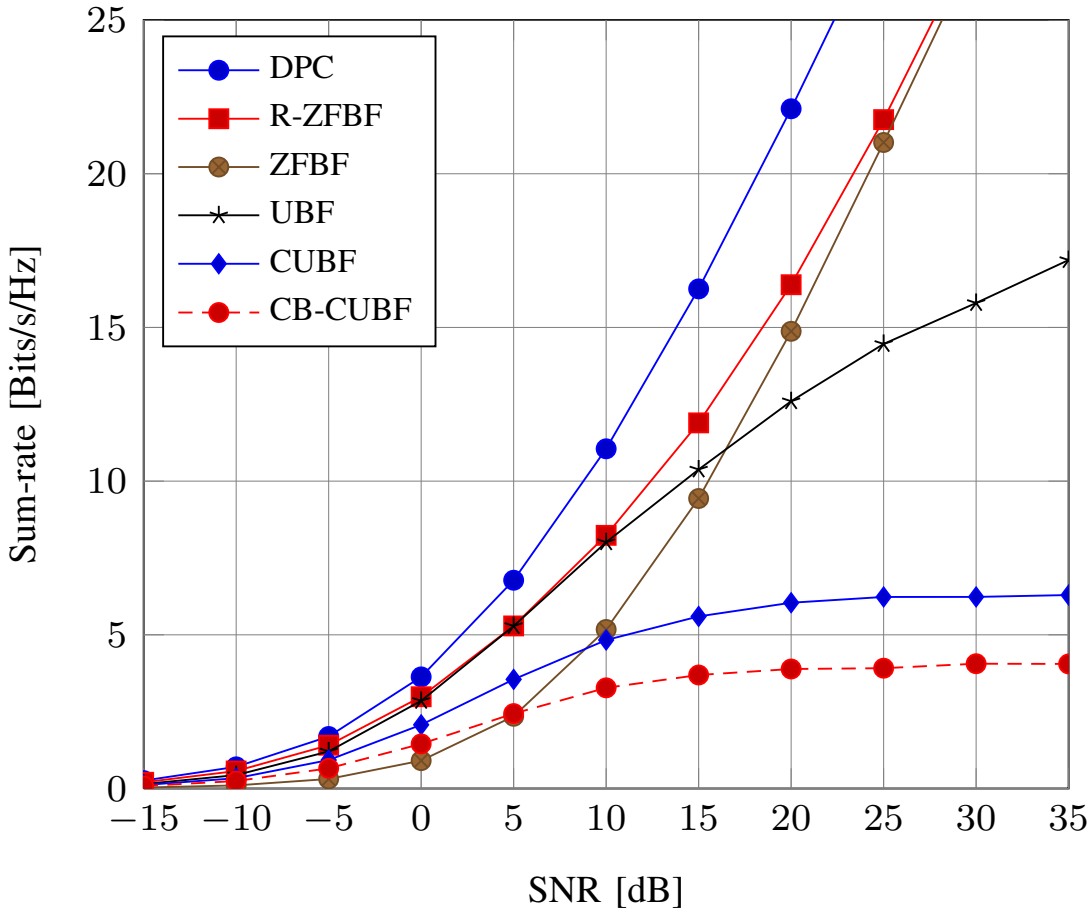

Figure 2: 4×4 MIMO independent Rayleigh fading channel, 1e4 channel realizations

5. Conclusions

This contribution evaluates various precoding techniques in terms of their achievable sum-rate on the MU-MIMO downlink. The main conclusions are as follows:
Optimal UBF is shown to outperform ZFBF for low to medium SNR but it increases the PAPR of the signal before the amplification. By imposing a constant modulus UBF the PAPR does not increase but the sum-rate reduces dramatically. However CUBF scheme shows a sum-rate gain of 40-50% compared to the LTE Rel-8 CB-UBF scheme. Hence, it can be noticed that by allowing the eNodeB to optimize the precoding based on CSI provided via feddback, the performance can be significantly increased.

A. Appendix

A.1. Zero-forcing beamforming

The criterion of ZFBF is to force the inter-user interference to zero. This can be achieved by performing an inversion of the channel matrix at the transmitter, i.e.

$$\mathbf{V}_{zf} = \frac{1}{\lambda}\mathbf{H}^{\mathsf{H}}\left(\mathbf{H}\mathbf{H}^{\mathsf{H}}\right)^{-1} \tag{9}$$

where $\frac{1}{\lambda} = \sqrt{\frac{1}{P}\mathrm{tr}\left((\mathbf{H}\mathbf{H}^{\mathsf{H}})^{-1}\right)}$ is the scaling factor to fulfill the sum-power constraint.

A.2. Regularized zero-forcing beamforming

A regularization factor can be introduced to the ZFBF in order to trade-off inter-user interference and noise power enhancement.

$$\mathbf{V}_{rzf} = \frac{1}{\lambda}\mathbf{H}^{\mathsf{H}}\left(\mathbf{H}\mathbf{H}^{\mathsf{H}} + \alpha\mathbf{I}\right)^{-1} \tag{10}$$

where $\frac{1}{\lambda}$ is chosen such that $\mathrm{tr}(\mathbf{V}_{rzf}\mathbf{V}_{rzf}^{\mathsf{H}}) = P$ and $\alpha = \frac{M\sigma_n^2}{P}$.

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