

**Source:** Ericsson  
**Title:** Complexity and Performance Improvement for Convolutional Coding  
**Agenda Item:** 5.5 Channel Coding  
**Document for:** Discussion and Decision

---

## 1. Introduction

In RAN1#48bis Malta, tail-biting convolutional coding with  $K=7$  [1] was adopted as a working assumption for LTE DL control channel while allowing further review on complexity and performance improvement techniques. To fulfill the design objective agreed in the way forward for DL control channel structure [2], a low-complexity and high-performing design based on a rate  $1/3$  code is needed.

We evaluate complexity and performance improvement for tail-biting convolutional coding. We find the decoding complexity can be reduced to roughly a quarter that of the  $K=9$  tailed code with simple decoder adaptation [4]. We further show the performance can be improved by adopting a  $K=7$  code with optimal distance spectrum [3]. At higher coding rates, the new design provides 0.2—0.3 dB gains over the previous tail-biting solution. At some of these rates, the  $K=9$  reference code does not perform better than the new  $K=7$  design over the information block size range of 32—160 bits.

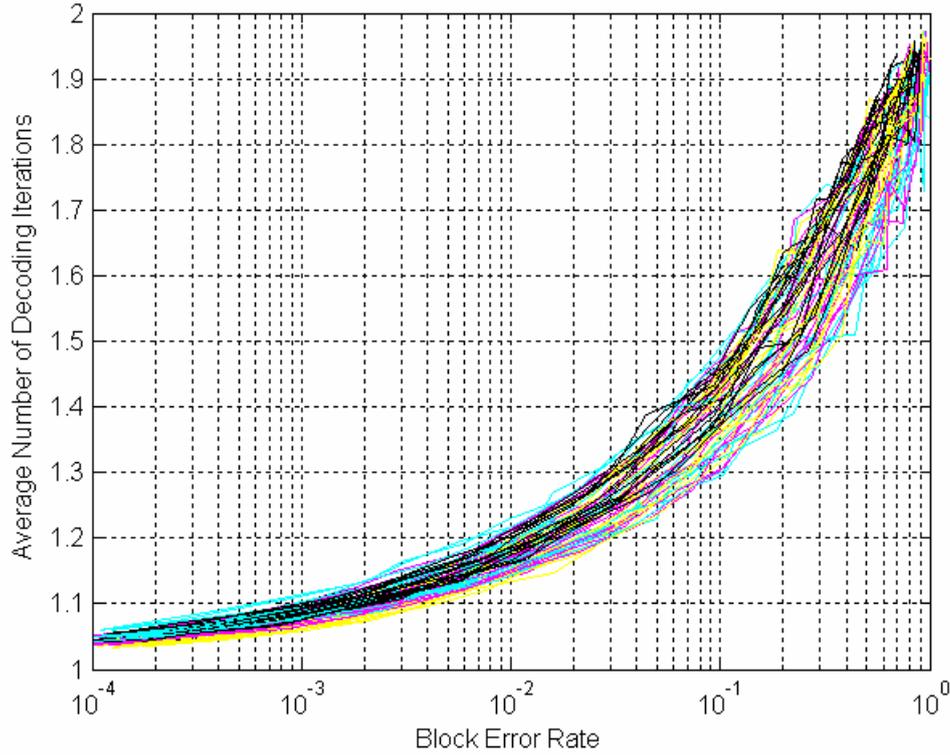
## 2. Decoding Complexity Analysis

Since the starting state of a tail-biting code is unknown to the decoder, wrap-around Viterbi decoding techniques on the circular tail-biting trellis are generally employed. In [1], the complexity of the tail-biting  $K=7$  code is estimated to be half of that of a  $K=9$  tailed code based on a fixed two-iteration decoding algorithm.

Academic research has shown that decoding complexity can, in fact, be further halved by executing the second iteration only if necessary [4]. A high-level description of this adaptive two-iteration algorithm is given below:

1. Start a 1<sup>st</sup> Viterbi recursion with equal state metrics.
2. If the most likely solution is tail-biting,  
    output the solution and stop decoding;  
    otherwise,  
    continue to step 3.
3. Start a 2<sup>nd</sup> Viterbi recursion with wrap-around state metrics.
4. If any tail-biting solution is found,  
    output the best tail-biting solution;  
    otherwise,  
    declare decoding failure.  
    Stop decoding.

This algorithm is applied to 85 different test configurations based on 17 information block sizes ( $B=32, 40, \dots, 160$ ) and 5 coding rates ( $r=1/3, 0.4, 0.5, 0.6, \text{ and } 0.7$ ). The average numbers of decoding iterations observed in the simulations are shown in Figure 1. It can be seen the average decoding complexity is largely determined by the operating block error rate (BLER). The impact of information block size or coding rate is rather minor. For a typical target BLER of 0.1%, it requires no more than 1.1 iterations on average. Furthermore, no performance losses are observed when comparing the BLER results in Section 3 with those provided in [1]. In summary, with simple adaptation, the decoding complexity of the  $K=7$  tail-biting code is roughly a quarter that of a  $K=9$  tailed code.



**Figure 1 Average decoding complexity at different operating BLER for K=7 tail-biting convolutional coding. The plot contains 85 curves corresponding to all combinations of 17 information block sizes (B=32, 40, ..., 160) and 5 coding rates (r=1/3, 0.4, 0.5, 0.6, and 0.7). Complexity of a K=9 tailed code is around 4 iterations of a K=7 code.**

### 3. Performance Improvement

The rate 1/3 codes selected for K=9 in [5] or for K=7 in [1] belong to the class of maximum free distance (MFD) codes developed more than 30 years ago [6][7]. In addition to free distances, however, the performance of convolutional coding depends also on the number of codewords (i.e., weights) at these distances. For instance, the BLER over a Rayleigh fading channel can be bounded by [7]

$$\text{BLER} \leq \sum_{d=d_f}^{\infty} \frac{a_d}{\sqrt{2\pi d}} \left( \overline{rE_b/N_0} \right)^{-d}, \quad (1)$$

where  $r$  is code rate and  $a_d$  is the number of codewords at distance  $d$ . It can be seen the BLER decays with the average SNR very slowly and the impact of the distance spectrum (i.e., the list of all  $a_d$ ) is significant. To minimize probabilities of errors, recent academic research has found codes with optimal distance spectra (ODS) [3], which, in addition to achieving maximum free distances, have lowest weights at all distances.

In the following, we investigate performance benefits of replacing the K=7 MFD code (with polynomials [133 145 175]) with the K=7 ODS code (with polynomials [133 171 165]). Performance of the tailed K=9 MFD codes (with polynomials [557 663 711] and [561 753]) is also included for reference. The tail-biting codes are decoded with the adaptive algorithm described in Section 2. The AWGN performance of these three codes at the native code rate (1/3) is compared in Figure 2. The ODS tail-biting code is found to provide small performance improvement over the MFD tail-biting code at this rate. Larger gains can be expected for fading channels.

The main purpose of the ODS code proposal, however, is not for the performance at the native rate but rather for the potential of improved puncturing performance. For the MFD codes, the rate 1/3 and rate 1/2 codes are specified by two different polynomial sets which are in general unrelated. Puncturing of a rate 1/3 code to a higher coding rate can thus be carried out over all coded bits uniformly. For the K=7 ODS codes [3], the polynomials for the rate 1/2 code ([133 171]) happen to be nested in those for the rate 1/3 code ([133 171 165]). This nesting structure can hence be utilized to improve punctured code performance. That is, coded bits produced by the polynomial [165] can be removed first while those from the polynomial [133] are always

retained. The actual puncturing patterns are otherwise uniform as those employed for the MFD codes. In Figures 3 and Figure 4, performance of this proposal is compared to the baseline tail-biting code and the reference tailed code. At higher coding rates, the new design provides 0.2—0.3 dB gains over the previous tail-biting solution. At higher rates, the K=9 reference code does not performs better than the new K=7 design over the entire information block size range of 32—160 bits.

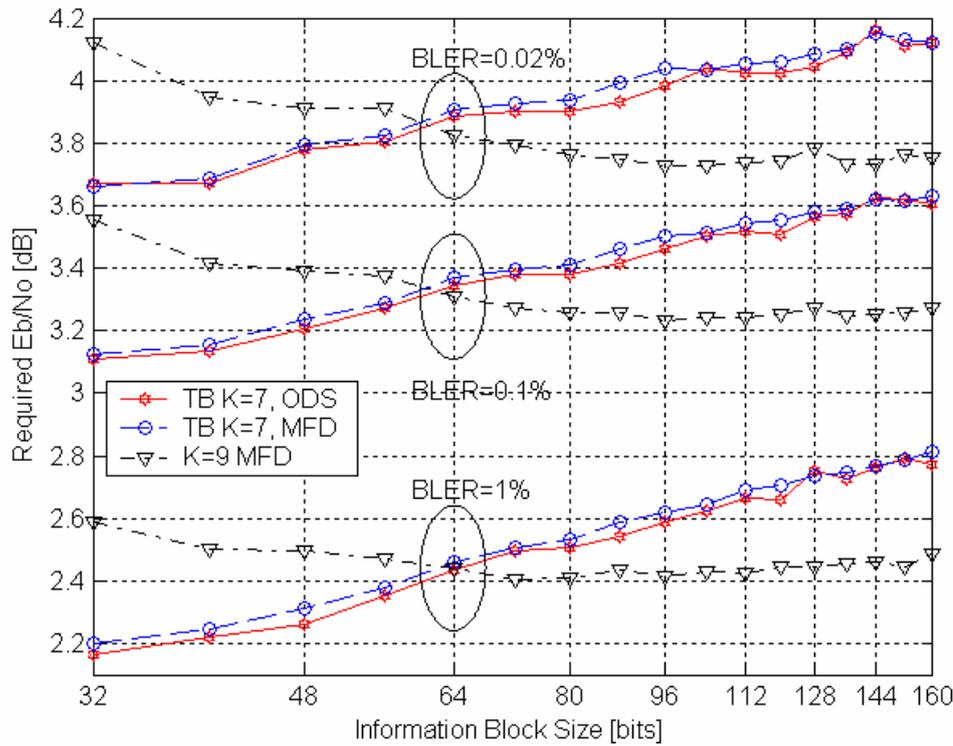


Figure 2 Performance for  $r=1/3$  coding with different block sizes and operating BLER targets.

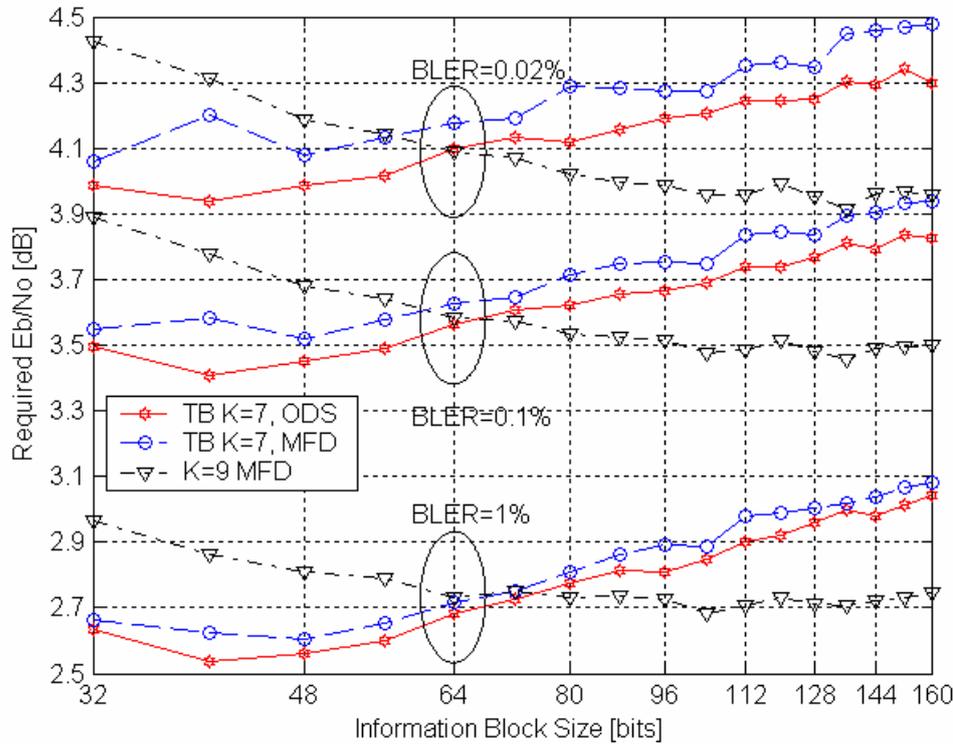


Figure 3 Performance for  $r=0.5$  coding with different block sizes and operating BLER targets.

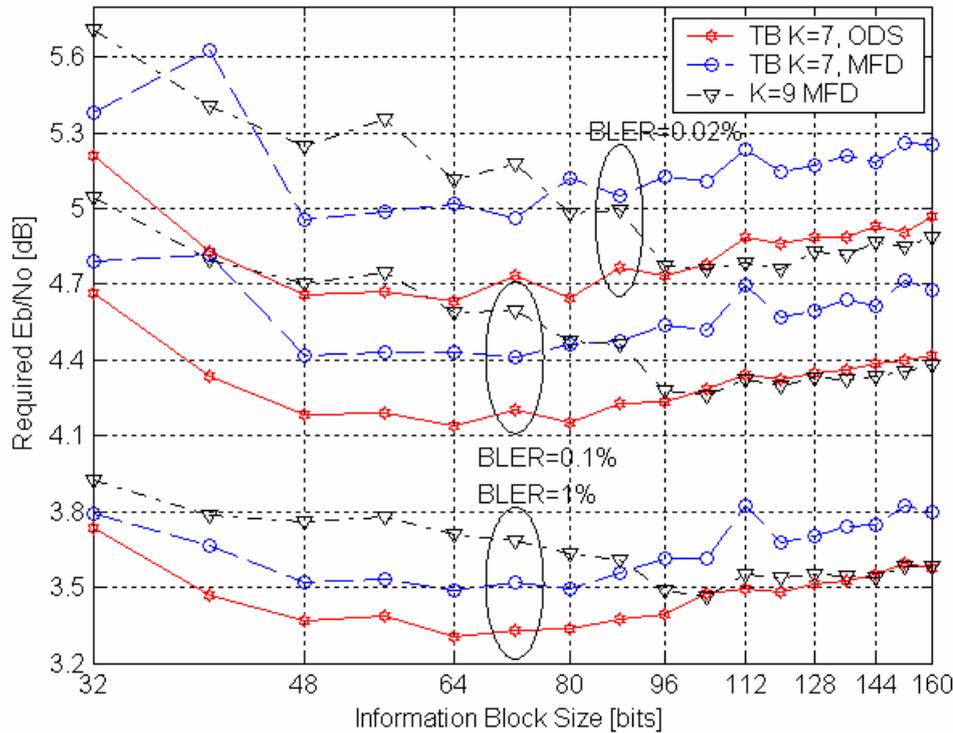


Figure 4 Performance for  $r=0.7$  coding with different block sizes and operating BLER targets.

## 4. Conclusion

We found the decoding complexity of  $K=7$  tail-biting codes can be reduced to roughly a quarter that of a  $K=9$  tailed code with simple decoder adaptation. We further demonstrated significant AWGN performance benefits of using a  $K=7$  code with optimal distance spectrum. Hence, we propose

- To adopt the  $K=7$  tail-biting convolutional code with polynomials [133 171 165] for control channels.

## 5. References

- [1] R1-071323, Motorola, "Performance of convolutional codes for the E-UTRA DL control channel," 3GPP TSG RAN WG1#48bis, St. Julians, Malta, Mar. 26 – 30, 2007.
- [2] R1-071820, Ericsson, Samsung, LGE, Panasonic, Nokia, NEC, Huawei, Nortel, Qualcomm, Motorola, NTT DoCoMo, Mitsubishi, Alcatel-Lucent, "DL control channel structure," 3GPP TSG RAN WG1#48bis, St. Julians, Malta, Mar. 26 – 30, 2007.
- [3] P. Frenger, P. Orten and T. Ottosson, "Convolutional codes with optimum distance spectrum," *IEEE Comm. Letters*, vol. 3, no. 11, pp. 317–319, Nov. 1999.
- [4] R. Y. Shao, S. Lin and M. Fossorier, "Two decoding algorithms for tailbiting codes," *IEEE Trans. on Comm.*, vol. 51, no. 10, pp. 1658–1665, Oct. 2003.
- [5] 3GPP TS 25.212 v6, "Multiplexing and Channel Coding (FDD) (Release 6)."
- [6] K. J. Larsen, "Short convolutional codes with maximal free distance for rates 1/2, 1/3, and 1/4," *IEEE Trans. Inform. Theory*, vol. IT-19, pp. 371–372, May 1973.
- [7] J. P. Odenwalder, *Optimal Decoding of Convolutional Codes*, Ph.D. dissertation, School Eng. Appl. Sci., Univ. California, Los Angeles, 1970.
- [8] J. G. Proakis, *Digital Communications*, 2<sup>nd</sup> ed., McGraw Hill, Inc., 1989.