3GPP TSG RAN WG1 #49 **R1-072406**
Kobe, Japan, May 7 – 11, 2007

Agenda Item: **7.5**

Source: **Broadcom**

Title: **Comparing rate matching puncturing to optimal periodic puncturing**

Document for: **Discussion/Decision**

1 Summary

In this document we firstly show by examples that the existing and proposed rate matching (RM) algorithms [1,2,3] generate the periodic puncturing patterns. However, those patterns are not generated based on the given mother turbo code. Therefore, we expect the performance loss. The question is how much loss those rate matching algorithm will have.

Theoretically if the period and final code rate are fixed there must be an optimal period puncturing pattern. However, it is not practical to find all puncturing patterns for all 188 QPP interleaves as well as all possible rates. Instead we use sub-optimal period puncture pattern that come from an optimal periodic puncture pattern for a single interleave.

In this document we compare the rate matching algorithms of Release 6 [1], circular buffering [2] and dithering with Release 6 puncturing (suggested in [3] for some puncturing rates) to the sub-optimal periodic puncturing method at rate 0.8 and 0.7.

Furthermore, we present a possible rate matching approach based on optimal or sub-optimal periodic puncturing.

2 Period puncturing and its generalization

Periodic Puncturing pattern: Since the turbo mother code is rate 1/3. Given L information bits $x_{0,0}, x_{0,1}, \cdots x_{0,L-1}$, there will be two parity sequences of L bits, say $x_{1,0}, \cdots, x_{1,L-1}$, $x_{2,0}, \cdots, x_{2,L-1}$ respectively. Suppose the puncturing period is q. then a puncturing pattern is given by 3 binary vectors of size q, namely

$$P_i = (e_{i,0}, e_{i,1}, \cdots, e_{i,q-1}), e_{i,j} \in \{0,1\}, i = 0,\ldots,2, j = 0,\ldots,q-1 \qquad \text{(EQ-1)}$$

The periodic puncturing based on the above puncturing pattern can be defined as the 3 sequences $y_{i,0}, y_{i,1}, \cdots y_{i,L-1}, i = 0,\ldots,2$, such that

$$y_{i,j} = \begin{cases} x_{i,j} & \text{if } e_{i,j \bmod q} = 1 \\ \delta & \text{if } e_{i,j \bmod q} = 0 \end{cases}, i = 0,1,2, j = 0,\ldots,L-1 \qquad \text{(EQ-2)}$$

Then the final output sequences after puncturing are those $y_{i,j}$ such that $y_{i,j} \neq \delta$.

On top of this, we can add 3 shifting numbers, say s_0, s_1, s_2. The punctured sequence with these shifting positions and the pattern given in (EQ-1) is

$$y_{i,j} = \begin{cases} x_{i,j} & \text{if } e_{i,j+s_i \bmod q} = 1 \\ \delta & \text{if } e_{i,j+s_i \bmod q} = 0 \end{cases}, i = 0,1,2, j = 0,\ldots,L-1 \qquad \text{(EQ-3)}$$

In the following we translate the puncturing algorithms in [1, 2, 3] to the periodic puncturing by examples

Example 1 Let information block size be1120 and final rate is 0.8. Consider the 2 different rate matching algorithms in [1, 2]. Although in the definition it not mentioned that the rate matching method is periodic, they can be translated to the periodic pattern as follows.

Release 6 rate matching [1]: The period is 8 and period pattern is
$P_0 = (11111111), P_1 = (00000001), \mathrm{P}_2 = (00000001)$ with shifting positions 0,3,6.

Circular buffer [2]: The period is 32 and period pattern is
$P_0 = (01111111111111110111111111111111)$;
$P_1 = (0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,1)$;
$P_2 = (0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,1)$.
To improve this, shifting may need be added.

Combined two periodic patterns: Suppose there are 2 different patterns for every sub-sequence, namely $P_{i,0}, P_{i,1}$ with periods $q_{i,0}$ and $\mathrm{q}_{\mathrm{i},1}$, respectively. Divide the whole sequence to $\mathrm{m_i}$ parts with sizes, $L_{i,0}, L_{i,1}$, such that $L = L_{i,0} + L_{i,1}$. Then the sequences can be represented as

$$x'_{i,0,0}, \cdots, x'_{i,0,L_{i,0}-1}, x'_{i,1,0}, \cdots, x'_{i,1,L_{i,1}-1}$$

such that $x'_{i,u,0}, x'_{i,u,1}, \cdots, x'_{i,u,L_u-1} = x_{i,M_u}, x_{i,M_u+1} \cdots, x_{i,M_u+L_u-1}$, where $M_0 = 0$ and $M_1 = L_{i,0}$. Use puncturing pattern $P_{i,u}$ to puncture $x'_{i,u,0}, x'_{i,u,1}, \cdots, x'_{i,u,L_u-1}$

Example 1 (continue) *Dithering with Release 6 puncturing:* Period and pattern for the information bits sequence are 1 and $P_o = (1)$, respectively. There are to two patterns for every parity bit sequence with $L_{1,0} = L_{2,0} = 4L/7, L_{1,1} = L - L_{1.0} = L_{2,0}$ and patterns are

$$P_{1,0} = P_{2,0} = (0\,0\,0\,0\,1\,0\,0\,0\,0\,1\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,1) \text{ of period } 21$$

and $P_{1,1} = P_{2,1} = (0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,1)$ of period 13.

3 Optimal periodic puncturing and sub-optimal combined period puncturing

3.1 Single optimal period

Since the sizes of all 188 QPP interleaves adopted in [4] are multiple of 8, we use one period, i.e. 8, for all interleaves. Since the mother code is rate 1/3, there are three output sequences, namely, $x_{0,0},\cdots,x_{0,L-1}$ $x_{1,0},\cdots,x_{1,L-1}$, $x_{2,0},\cdots,x_{2,L-1}$. We start with finding the optimal puncturing pattern for rate $r = 8/D$, $\mathrm{D} = 9,10,11,\ldots,23$, i.e

$$\begin{aligned} &8/9,8/10,8/11,8/12 = 2/3,8/13,8/14 = 4/7,8/15,8/16 = 1/2, \\ &8/17,8/18 = 4/9,8/19,8/20 = 2/5,8/21,8/22 = 4/11,8/23. \end{aligned} \qquad \text{(EQ-4)}$$

The idea way to approach the periodic optimal is to find optimal period 8 patterns for all rates in (EQ-4) and for all 188 QPP interleaves. A sub-optimal choice is to find an optimal pattern for a cistern interleave and then use it for all other interleave .

For example, on rate 8/10=0.8, we found an optimal pattern for QPP interleave size 1120, i.e.

$$P_0 = (11101111),\ P_1 = (01010000),\ P_2 = (00001000) \qquad \text{(EQ-5)}$$

The following figure shows the big improvement to the algorithm [1,2] and dithering with Release 6 punctureing on QPSK AWGN channel with 6 iterations.

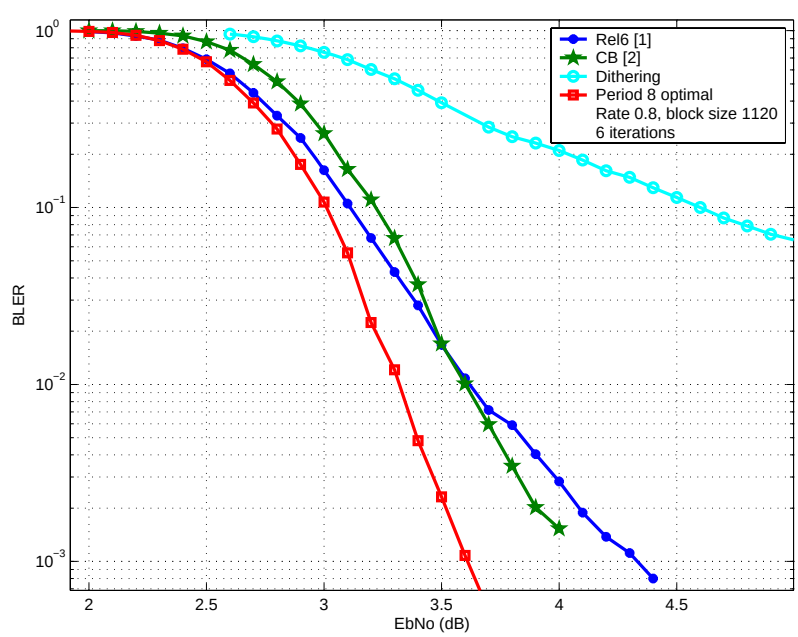

Figure 1 Interleave size 1120, rate 0.8, 6 iterations QPSK

The following 3 figures show the improvement by using this optimal pattern for all 188 QPP interleaves for rate 0.8 on block error rate (BLER), 0.1, 0.01 and 0.002.

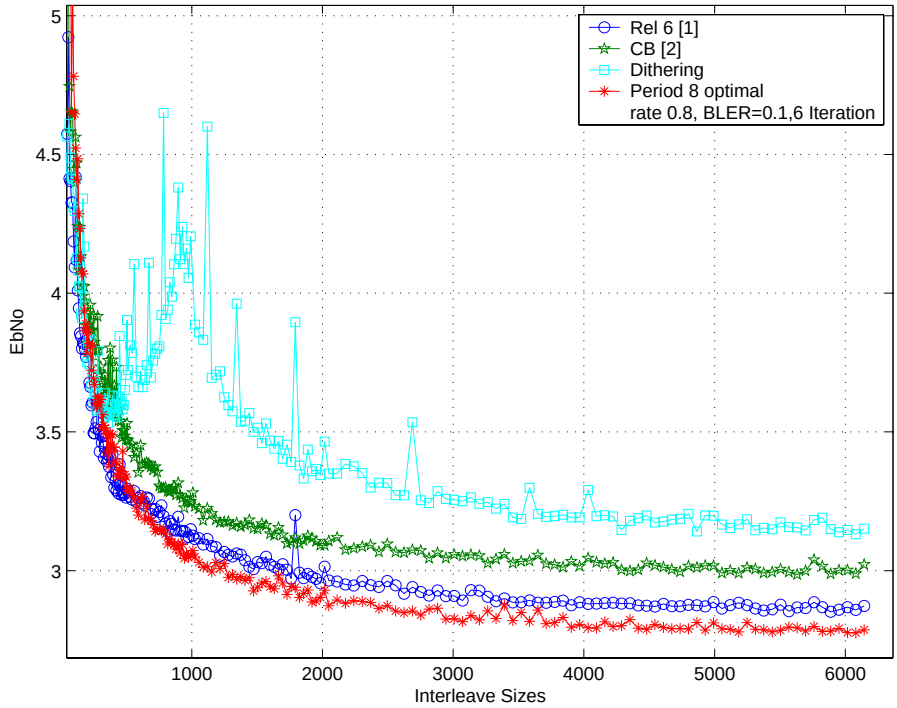

Figure 2 Rate 0.8 BLER=0.1, QPSK

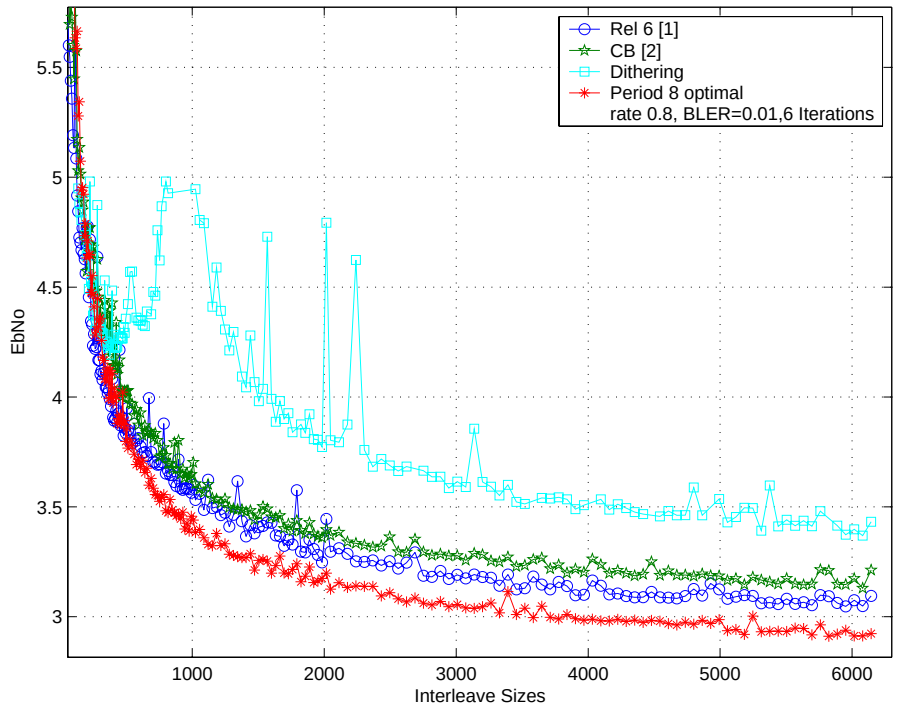

Figure 3 Rate 0.8, BLER =0.01,QPSK

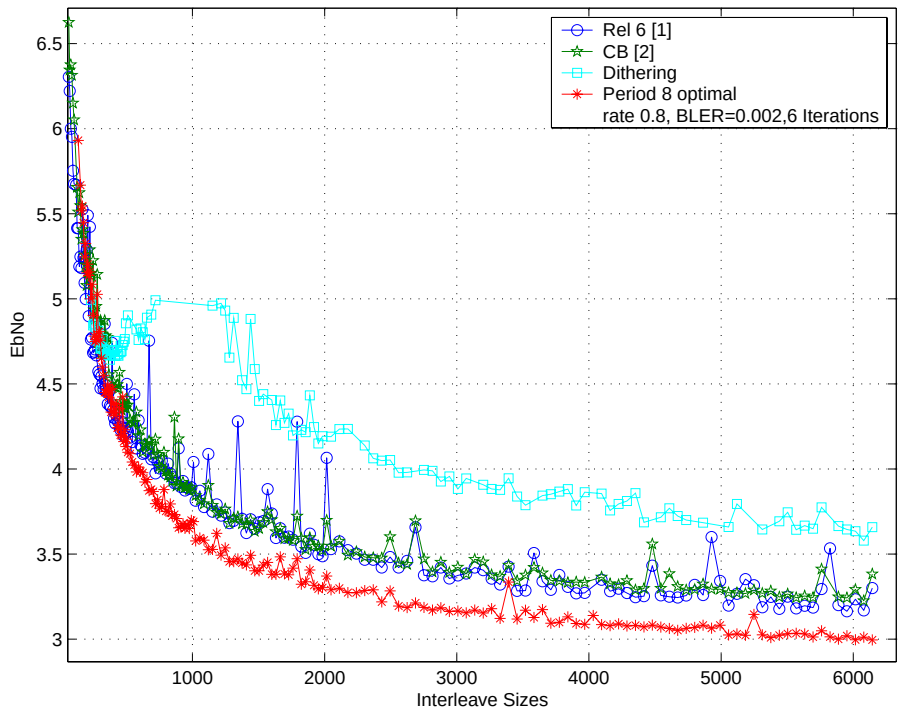

Figure 4 Rate 0.8, BLER = 0.002, QPSK

From the three pictures we can see that the sub-optimal pattern of period 8 out perform [1,2] and dithering method suggested for some rates by [3]. Moreover, this method gives no bad peaks.

3.2 Combining several period patterns

To have puncturing pattern for the rate not among the rates in (EQ-4), we can combine patterns from the rates in (EQ-4). To find out the percentage of different periods are needed we can use the following formula. Suppose the attempted code rate is r and we want use t rates $R_0 = 8/D_{j_0}, R_1 = 8/D_{j_1}, \cdots, R_{t-1} = 8/D_{j_{t-1}}$ in (EQ-4). The percentage for every rate are denoted by $a_0, a_1, \ldots, a_{t-1}$ and must satisfy

$$\frac{1}{\sum_{u=0}^{t-1} a_u / R_u} = r \qquad \text{(EQ-6)}$$

such that $a_0 + a_1 + \cdots + a_{t-1} = 1$.

In particular we choose t=2. Then $a_1 = 1 - a_0$ and $\dfrac{1}{\dfrac{a_0}{R_0} + \dfrac{(1-a_0)}{R_1}} = r$.

Let $L = 8\tilde{L}$. We can find integers c_j, C such that $a_i = {c_i}/{C}$. Let

$$\tilde{L}_0 = \left\lceil \frac{c_o \tilde{L}}{C} \right\rceil \text{and } \tilde{L}_1 = \tilde{L} - \tilde{L}_0 .$$

Let "cont_P" be a counter on the number of periods. Now the puncturing method is as follows:

If $cont_p \bmod C < c_0$, use puncturing pattern P_0 , otherwise use puncturing pattern P_1

Once again we find an optimal patterns for interleave size 1120. Consider rate r=0.7. Since this rate is between 8/11 and 8/12, we combine optimal R_1=8/11 puncturing pattern

$$P_{0,0} = (1\,0\,1\,1\,1\,1\,1\,1\,1), P_{0,1} = (0\,1\,0\,1\,0\,0\,0\,1), P_{0,2} = (0\,0\,0\,0\,0\,1\,0\,1)$$

And the optimal R_2=8/12=2/3 puncturing pattern:

$$P_{1,0} = (1\,1\,1\,1\,0\,1\,1\,1), P_{1,1} = (1\,0\,1\,0\,1\,0\,0\,0), P_{1,2} = (0\,0\,0\,0\,1\,0\,0\,1)$$

Since we combine two patterns, we have $a_1 = 1 - a_0$ and (EQ-6) becomes

$$\frac{1}{\frac{a_0}{R_0} + \frac{(1-a_0)}{R_1}} = 0.7$$

Thus, $a_0 = 4/7$ and $a_1 = 3/7$.

The following figure shows the improvement to [1,2] and dithering algorithm on QPSK AWGN channel with 6 iterations on size 1120

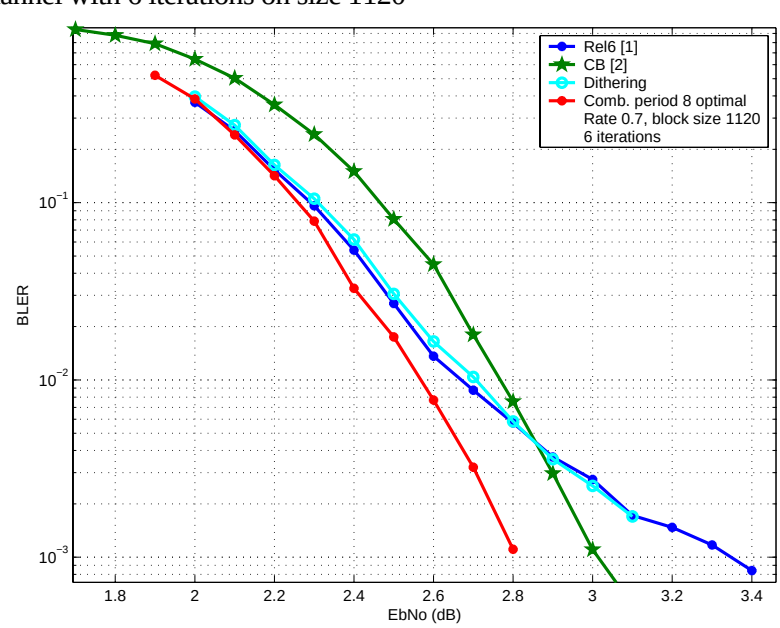

Figure 5 Rate 0.7, QPP size 1120, QPSK, 6 iterations

The following 3 figures show the performance over 188 QPP interleaves for rate 0.7 outputs.

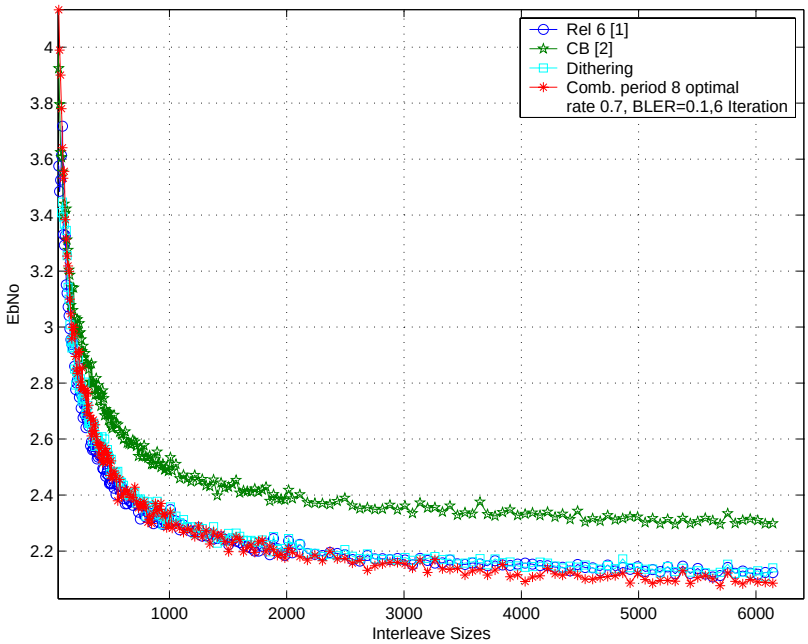

Figure 6 Rate 0.7, BLER=0.1, 6 iterations, QPSK

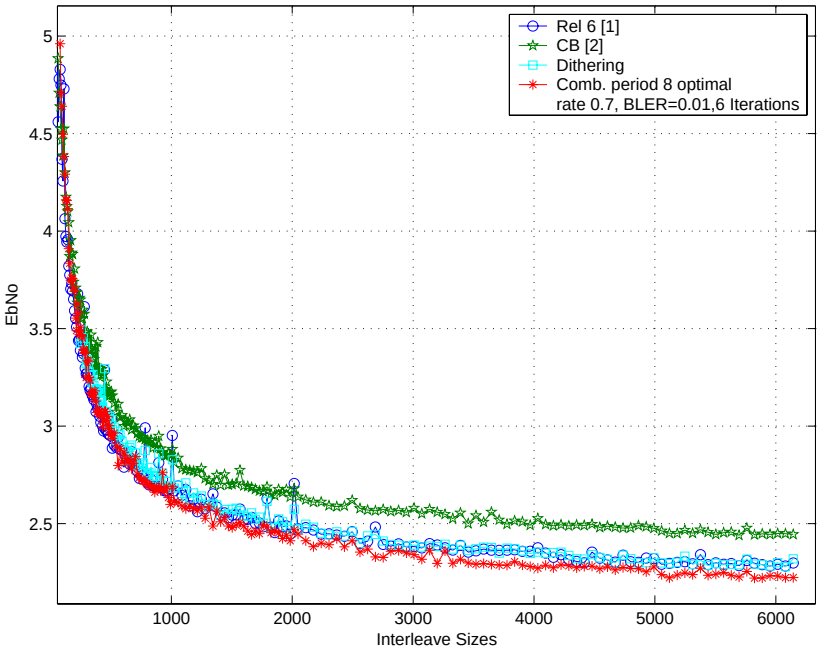

Figure 7 Rate 0.7, BLER=0.01, 6 iterations

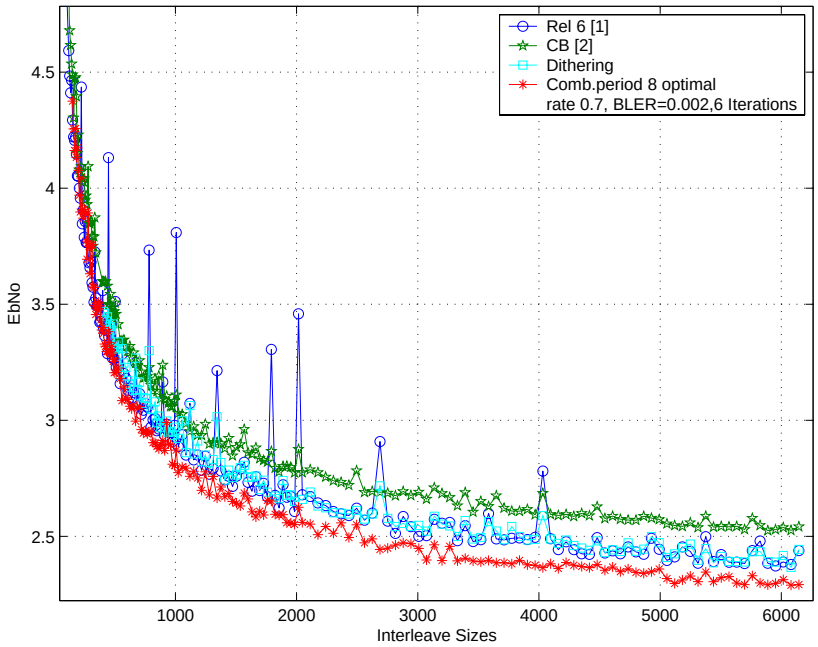

Figure 8 Rate 0.7, BLER=0.002, 6 iterations, QPSK

From the three pictures we can see that the sub-optimal combined pattern of period 8 out perform [1,2] and dithering method suggested for some rates by [3]. Moreover, this method gives no bad peaks.

3.3 On tail bits

Turbo encoder adopted by 3GPP LTE is with 12 bits extra tail bits added after coded bits. Therefore, to make the final output code rate as needed one need puncture additional 12 bits inside the code block. One way to do this is to evenly distribute 12 positions on the not-yet-punctured positions and puncture them before send out with 12 tail bits at end.

4 Reference

[1] Technical Specification Group Radio Access Network; Multiplexing and channel coding (FDD) (Release 6), 3GPP TS 25.212 V6.8.0

[2] Ericsson, LGE, Motorola, Qualcomm, Samsung and ZTE, “Circular buffer rate matching”, E-mil proposal on reflector: 3GPP_TSG_RAN_WG1@LIST.ETSI.ORG, 4/14/2007

[3] Nokia Siemens Networks, Nokia, “Description of the Dithering Algorithm for LTE Rate Matching,” E-mil proposal on reflector: 3GPP_TSG_RAN_WG1@LIST.ETSI.ORG, 4/15/2007

[4] Technical Specification Group Radio Access Network: Multiplexing and channel coding (release 8), (Draft) 3GPP TS 36.212 V0.4.2 (2007-02)