# A New Comma Free Code Scheme for TDD Synchronization 

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#### Abstract

The current UTRA TDD scheme uses a $(16,3)$ CFC, which was designed for the UTRA FDD mode and is capable of identifying up to sixteen different phases of the Frame timing. This $(16,3)$ code is decimated by two and the resulting length 8 code is stretched out in time over 4 Frames. The $(16,3)$ code has been optimized for the FDD mode. Consequently, its Stage 2 performance is poorer as compared to the Modulated SSC method. In this contribution, we propose to use a length 2 comma free code (CFC) for the $2^{\text {nd }}$ step of cell search procedure for the UTRA TDD mode. This length 2 CFC makes use of six orthogonal length 256 secondary synchronization codes (SSC's). Thus, its complexity is similar to that of the recent Modulated SSC scheme proposed by IDC, while its performance is significantly better.


## 1 Introduction

In a recent submission IDC proposed the use of 6 BPSK modulated SSC's for the second step of synchronization in the UTRA TDD mode [1]. This scheme is shown to improve the detection performance over the cell search procedure using a $(16,3)$ CFC $[2-4]$ and the hierarchical code method [5].

The $(16,3)$ CFC was originally proposed and optimized for the UTRA FDD mode, where the synchronization channel is transmitted in every slot. The length 16 of the CFC comes from the fact that there are 16 slots in a frame and therefore using the ( 16,3 ) CFC all cyclic shifts (and thus the 16 possible frame timing phases) are uniquely identified. Thus, using the ( 16,3 ) CFC, decimated by 2 and transmitted over 8 frames is not optimal. Consequently, it performs poorer as it performs one out 256 decision while the Modulated SSC method performs a one out of 64 decision.

In this contribution we propose a length 2 CFC which is designed to identify only two cyclic shifts. This CFC is designed using an alphabet size of 24. The alphabet is formed using 6 SSC's that are QPSK modulated (modulated by $\pm 1$ and $\pm \mathrm{j}$ ). Note that such a modulation is possible, as the actual modulation on the WCDMA downlink is QPSK. The BPSK modulation proposed by IDC is obtained by using only two points of the QPSK constellation. Using this new alphabet, a total of 60 Comma Free codewords are possible, while only 32 codewords corresponding to the 32 code groups are used. Simulation results show that its performance is significantly better than the Modulated SSC method while its complexity is similar to that of the Modulated SSC method.

## 2 The Length 2 Comma Free Code

Consider the sequences $\left\{\mathrm{s}_{1}, \mathrm{~s}_{2}, \ldots, \mathrm{~s}_{\mathrm{N}}\right\}$. A simple length 2 comma free code can be formed as $\left\{\left(s_{\mathrm{i}}+\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2,\left(\mathrm{~s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2\right\} \mathrm{i}=1: \mathrm{N}-1, \mathrm{k}=\mathrm{i}+1: \mathrm{N}$. It is easy to verify that this code design results in $\mathrm{N}(\mathrm{N}-1) / 2$ Comma free codewords. Thus, choosing $N=6$, we obtain 15 Code words. Since a coherent detection can be performed, we also allow codewords of the form $\left\{-\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2\right.$, $\left.-\left(\mathrm{s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2\right\}$, $\left\{\mathrm{j}\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{j}}\right) / \sqrt{ } 2, \mathrm{j}\left(\mathrm{s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{j}}\right) / \sqrt{ } 2\right\}$ and $\left\{-\mathrm{j}\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{j}}\right) / \sqrt{ } 2,-\mathrm{j}\left(\mathrm{s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{j}}\right) / \sqrt{2} 2\right\}, \mathrm{i}=1: \mathrm{N}-1, \mathrm{j}=\mathrm{i}+1: \mathrm{N}$. It is easy to see that the resulting set of 60 codewords are Comma Free. Thus, with N spreading sequences, we can form $2 \mathrm{~N}(\mathrm{~N}-1)$ Comma Free Codewords. It is
easy to verify that the minimum distance of these codewords is $\sqrt{ }$, while that of Modulated SSC method (over one frame) is $2 / \sqrt{3}$. Thus, the length 2 Comma Free Sequences have a better minimum distance and therefore should perform better than the Modulated SSC method. That this is indeed the case is verified by the simulation results presented in Figures 1 through 9.

Note that only 32 codewords need to be chosen out of the 60 possible codewords. These 32 codewords are chosen to be 30 codewords formed as $\pm\left\{\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{k}}\right) / \sqrt{2}\right.$, $\left.\left(\mathrm{s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{k}}\right) / \sqrt{2}\right\} \mathrm{i}=1: 5, \mathrm{k}=\mathrm{i}+1: 6$ and two codewords $\pm\left\{\mathrm{j}\left(\mathrm{s}_{1}+\mathrm{s}_{2}\right) / \sqrt{ } 2,\left(\mathrm{~s}_{1}-\mathrm{s}_{2}\right) / \sqrt{2} 2\right\}$.

## 3 Complexity Comparison

The complexity of Stages 1 and 3 of acquisition is the same for both the length 2 CFC method and the Modulated SSC methods. We therefore compare only the second stage complexity.

As derived in [1], 32 correlations are obtained performing length 8 correlations and the Fast Hadamard Transform is applied to these correlation values to obtain the correlations with the 6 SSC's and the PSC, requiring $8 \times 32+2 \times 16 \times \log _{2} 16+7=391$ complex additions.

Again, the phase of the correlation with the PSC is used as a reference for the correlations with the 6 SSCs. This requires (from [1]) 28 real multiplications and 13 real additions.

There are 64 possible combinations of the 6 SSC's $\{+1,+1,+1,+1,+1,+1\}$ to $\{-1,-1,-1,-1,-1,-1\}$ for the Modulated SSC method. Noting that some of these combinations are simply negatives of others and using other redundancies, the 64 combinations require 54 real additions. Averaging the 64 decision variables over K slots requires approximately 64 real additions per slot for large K (actually it is 64 ( K $1) / \mathrm{K}$ additions). Selecting the maximum after averaging over K slots requires $\left(\log _{2} 64\right) / \mathrm{K}$ compares per slot. This number is the same for both the CFC and the Modulated SSC methods and for large K is very small. So we neglect it in our analysis. Thus, for the modulated SSC method, we require 118 real additions per slot for computing, averaging the decision variables and selecting the maximum.

Now for the CFC, we note that multiplication by j is simply flipping the imaginary and real parts. Also, as noted above using $\pm\left\{\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{j}}\right) / \sqrt{ } 2\right.$, $\left.\left(\mathrm{s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{j}}\right) / \sqrt{ } 2\right\}$ produces 30 codewords and using $\pm \mathrm{j}\left\{\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{j}}\right) / \sqrt{ } 2\right.$, $\left.\left(\mathrm{s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{j}}\right) / \sqrt{ } 2\right\}$, $\mathrm{i}=1 \ldots 5, \mathrm{j}=\mathrm{i}+1 \ldots 6$ produces 30 more codewords. Since we require only 32 codewords, we use all the 30 codewords obtained from $\pm\left\{\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{j}}\right) / \sqrt{ } 2\right.$, $\left.\left(\mathrm{s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{j}}\right) / \sqrt{2}\right\}, \mathrm{i}=1 \ldots 5, \mathrm{j}=\mathrm{i}+1 \ldots 6$ and the two codewords $\pm j\left\{\left(s_{1}+\mathrm{s}_{2}\right) / \sqrt{ } 2,\left(\mathrm{~s}_{1}-\mathrm{s}_{2}\right) / \sqrt{2}\right\}$. Again noting that some of these combinations are simply negatives of the others, we require 32 additions per slot to obtain the 64 decision variables. In order to average the 64 decision variables over K slots, we require approximately 64 real additions per slot for large K (actually it is $64(\mathrm{~K}-1) / \mathrm{K}$ additions). Again neglecting the compares required for selecting the maximum, which is the same for the modulated SSC method, for the length 2 CFC scheme, we require 96 real additions per slot for computing, averaging the decision variables and selecting the maximum.

Table 1 shows the complexity comparison of the Modulated SSC scheme and the length 2 CFC.

| Method | Complexity at each SCH slot |  |
| :---: | :---: | :---: |
|  | Real Multiplications | Real Additions |
| Length 2 CFC | 28 | 891 |
| Modulated SSC | 28 | 913 |

Table 1. Table showing the complexity of the length 2 CFC method and the Modulated SSC method for the 32 long code groups case. The table shows that both methods have very similar complexity for this case.

## 4 Simulations and performance

As stage 1 performance of both the Comma Free Code (CFC) method and that of the modulated SSC scheme are the same, only the stage 2 performance results are shown under various conditions and number of slots used in the second stage. Similar to the assumptions made in TSGR1\#5(99)578, it was assumed that the synchronization channel is sent in the first and the eighth slot and that $\mathrm{T}_{\text {gap }}=-256$. Also, it was assumed that the correct position of the secondary synchronization channel is obtained from the $1^{\text {st }}$ stage of the acquisition. The probability of error in stage 2 versus the Chip-to-Noise ratio (CNR) of the secondary synchronization channel ( SCH ) is plotted in Figures 1 through 6. In all these simulation results, 32 long code groups and the Comma Free codes corresponding to the groups using 6 SSC's as described above were used. Figure 1 indicates the AWGN performance while the other figures indicate the performance in Rayleigh channels with different Doppler rates. It can be seen from these figures that the length 2 Comma Free Code performs about 1.0dB better than the Modulated SSC method.

It is very straightforward to extend the length 2 CFC concept to the case where there are 256 groups. This case occurs when one has to identify the slot number of the CCPCH in addition to the frame timing. In such a case, we require 12 SSC's to represent the 256 groups. The Modulated SSC method requires 9 codes. The complexity for each method in this case is listed in Table 2, which shows that the complexities of both the length 2 CFC and the Modulated SSC methods are similar. However, the minimum distance for the CFC is still maintained at $\sqrt{ }$, while that of the modulated SSC method decreases to $2 \sqrt{ } 2 / 3$. This results in an improvement in performance of the length 2 CFC method over the modulated SSC method of about 2.0 dB . The simulation results for this case are plotted in figures 7 and 8.

| Method | Complexity at each SCH slot for the 256 long code groups case |  |
| :---: | :---: | :---: |
|  | Real Multiplications | Real Additions |
| Length 2 CFC | 52 | 1593 |
| Modulated SSC | 40 | 1725 |

Table 2. Table showing the complexity of the Length 2 CFC and the Modulated SSC schemes for the 256 long code group case. The table shows that both the methods have similar complexity.

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Figure 1. Figure comparing the Stage 2 performance of the Length 2 CFC to that of the Modulated SSC method for the AWGN case. The figure shows that the proposed method performs about 1.0 dB better than the Modulated SSC method. 8 slots were used in Stage 2.

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Figure 2. Figure comparing the Stage 2 performance of the length 2 CFC with that of the Modulated SSC scheme for the Rayleigh fading case. The figure shows that the performance of the length 2 CFC is better than that of the modulated SSC method by about 1.0 dB for both 4 and 8 slots case. This is because the length 2 CFC has better distance than the modulated SSC method.

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Figure 3. Figure comparing the Stage 2 performance of the length 2 CFC with that of the Modulated SSC scheme for the Rayleigh fading case under a 6 KHz Frequency error. The figure shows that the performance of the length 2 CFC is still better than that of the modulated SSC method by about 1.0 dB for both 4 and 8 slots case.

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Figure 4. Figure comparing the Stage 2 performance of the length 2 CFC with that of the Modulated SSC scheme for the single path Rayleigh fading case, with a Doppler of 100 Hz . There is no Frequency error. The figure shows that the performance of the length 2 CFC is still better than that of the modulated SSC method by about 1.0 dB for both 4 and 8 slots case.

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Figure 5. Figure comparing the Stage 2 performance of the length 2 CFC with that of the Modulated SSC scheme for the single path Rayleigh fading case, with a Doppler of 100 Hz . The Frequency error is 6 KHz . The figure shows that the performance of the length 2 CFC is still better than that of the modulated SSC method by about 1.0 dB for both 4 and 8 slots case.

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Figure 6. Figure comparing the Stage 2 performance of the length 2 CFC with that of the Modulated SSC scheme for the single path Rayleigh fading case, with Doppler's of 5 Hz and 460 Hz . The Frequency error is 6 KHz and the number of slots was 4 . The figure shows that the performance of the length 2 CFC is still better than that of the modulated SSC method by about 1.0 dB .

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Figure 7. Figure comparing the Stage 2 performance of the length 2 CFC with that of the Modulated SSC scheme for the AWGN case. The number of long code groups is 256 . The figure shows that the performance of the length 2 CFC is better than that of the modulated SSC method is greater than 1.5 dB .

Figure 8. Figure comparing the Stage 2 performance of the length 2 CFC with that of the Modulated SSC scheme for the single path Rayleigh fading case, with Doppler's of 100 Hz and 460 Hz . There is no Frequency error and the number of slots was 4 . The figure shows that the performance of the length 2 CFC is still better than that of the modulated SSC method by about 2.0 dB .

## 4 Conclusions and Extensions

1) The $(16,3)$ CFC decimated by 2 , which is currently being used for the TDD mode of WCDMA is not optimal as the original $(16,3)$ code was optimized for the FDD mode.
2) The length two Comma Free Code designed using six SSCs which are QPSK modulated can support up to 60 groups and have significantly better performance and similar complexity as compared to the Modulated SSC method.
3) It is easy to extend the length two Comma Free Codes to support up to 256 groups. This case occurs when not only the frame timing but also the position of the CCPCH has to be determined. Even for this case, the complexity of the CFC scheme is similar to that of the Modulated SSC method, while its performance is significantly better.
4) In addition to the frame timing if the start of the CCPCH frame identification is also desired, it is easy to extend the length 2 CFC to a length 4 CFC with similar distance. One easy way to do it would be to use codewords of the form $\pm\left\{\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{k}}\right) / \sqrt{2}\right.$, $\left(\mathrm{s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{k}}\right) / \sqrt{2}$, $\left.\left(\mathrm{s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2,\left(\mathrm{~s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{k}}\right) / \sqrt{2}\right\}$ and $\pm \mathrm{j}\left\{\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{k}}\right) / \sqrt{2} 2,\left(\mathrm{~s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2,\left(\mathrm{~s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{k}}\right) / \sqrt{2} 2,\left(\mathrm{~s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{k}}\right) / \sqrt{2}\right\}$.
5) If coherent detection is to be used, $\mathrm{T}_{\text {gap }}>0$ is possible as long as the $\mathrm{T}_{\text {gap }}$ the is such that the phase change, due to the Doppler and the frequency error, between the PSC and the SSC is not very high. If this change is too high, then a non-coherent scheme has to be employed. In such a case, we will be able to use length 2 CFC's of the form $+\left\{\left(\mathrm{s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2,\left(\mathrm{~s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2,\left(\mathrm{~s}_{\mathrm{i}}-\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2,\left(\mathrm{~s}_{\mathrm{i}}+\mathrm{s}_{\mathrm{k}}\right) / \sqrt{ } 2\right\}$. Thus there is a reduction in the number of groups which can be supported using the same number of SSC's (N). The number of SSC's (N) should be increased for this case.

## References

[1] A BPSK Modulated Secondary Synchronization Code based Cell Search in UTRA TDD, TSGR1\#5(99)578, Interdigital Communications Corporation.
[2] Comma Free Codes for Fast PN Code Acquisition in WCDMA Systems: A proposal for the UTRA Concept, Tdoc SMG2 UMTS-L1 72/98, Texas Instruments.
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