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**Title: Golay-Hadamard Sequence Based RACH Preamble Design for Large Cell (Part-I *Algorithm*)**

**Source: Nortel Networks<sup>1</sup>**

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**0.0 Summary**

In this contribution, we present an interleave concatenation of Golay and Hadamard sequence to construct the RACH preamble and signatures for 3GPP in large cell operation.

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<sup>1</sup> Contact Person: Wen Tong, Evelyne Le Strat, Nortel Networks  
e\_mail: wentong@nortelnetworks.com

# 1.0 Golay Sequence Based RACH Signature

## 1.1 Introduction

The current working assumption on RACH preamble is based on a 4096 Gold code (preamble spreading code) concatenating with a set of 16 orthogonal Gold sequence with 16-symbol length (preamble signature). In Ref [1], it was shown such a RACH preamble has a poor aperiodic auto-correlation, with a high sidelobes, instead, the Golay pair sequence was proposed as RACH preamble spreading code and along with a special designed signatures. In Ref [2], a Long PN code of 4096 chips was proposed to replace the Gold preamble spreading code in order to share the same RACH detector hardware with the message decoding part despreaders. More significantly, In Ref [3], it was shown that current working assumption and the Golay based RACH preamble exhibit a high inter-signature cross correlation peak when the uncertain delay is multiple of the 256 chips. In Ref[4], the Long PN code is used as preamble spreading code as a fix to such a problem and a new design of Hadamard signature is proposed to allow good robustness to combat high Doppler shift and frequency offset. In this contribution, by extension construction of long Golay sequence, we present a Golay-Hadamard based RACH preamble, with better correlation properties and simplified hardware complexity

## 1.2 Ericsson Proposal

In order to simplify the 1<sup>st</sup> stage 256 chip match filter, and improve the aperiodic auto-correlation of RACH signature, Ericsson proposed to use Golay complementary pair sequence instead of Gold sequence based on the fact that such a match filter can be replaced by so-called Efficient Golay Correlator (EGC). Table 1 lists of 16 signatures which consists of constituent Golay complementary pair sequence A and B.

TABLE 1. Ericsson Signature

| Signature | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|-----------|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1         | A | A  | B  | B  | A  | -A | -B | B  | A  | -A | B  | -B | A  | A  | -B | -B |
| 2         | A | A  | B  | B  | A  | -A | -B | B  | -A | A  | -B | B  | -A | -A | B  | B  |
| 3         | A | -A | B  | -B | A  | A  | -B | -B | A  | A  | B  | B  | A  | -A | -B | B  |
| 4         | A | -A | B  | -B | A  | A  | -B | -B | -A | -A | -B | -B | -A | A  | B  | -B |
| 5         | A | A  | B  | B  | -A | A  | B  | -B | A  | -A | B  | -B | -A | -A | B  | B  |
| 6         | A | A  | B  | B  | -A | A  | B  | -B | -A | A  | -B | B  | A  | A  | -B | -B |
| 7         | A | -A | B  | -B | -A | -A | B  | B  | A  | A  | B  | B  | -A | A  | B  | -B |
| 8         | A | -A | B  | -B | -A | -A | B  | B  | -A | -A | -B | -B | A  | -A | -B | B  |
| 9         | A | A  | -B | -B | A  | -A | B  | -B | A  | -A | -B | B  | A  | A  | B  | B  |
| 10        | A | A  | -B | -B | A  | -A | B  | -B | -A | A  | B  | -B | -A | -A | -B | -B |
| 11        | A | -A | -B | B  | A  | A  | B  | B  | A  | A  | -B | -B | A  | -A | B  | -B |
| 12        | A | -A | -B | B  | A  | A  | B  | B  | -A | -A | B  | B  | -A | A  | -B | B  |
| 13        | A | A  | -B | -B | -A | A  | -B | B  | A  | -A | -B | B  | -A | -A | -B | -B |
| 14        | A | A  | -B | -B | -A | A  | -B | B  | -A | A  | B  | -B | A  | A  | B  | B  |
| 15        | A | -A | -B | B  | -A | -A | -B | -B | A  | A  | -B | -B | -A | A  | -B | B  |
| 16        | A | -A | -B | B  | -A | -A | -B | -B | -A | -A | B  | B  | A  | -A | B  | -B |

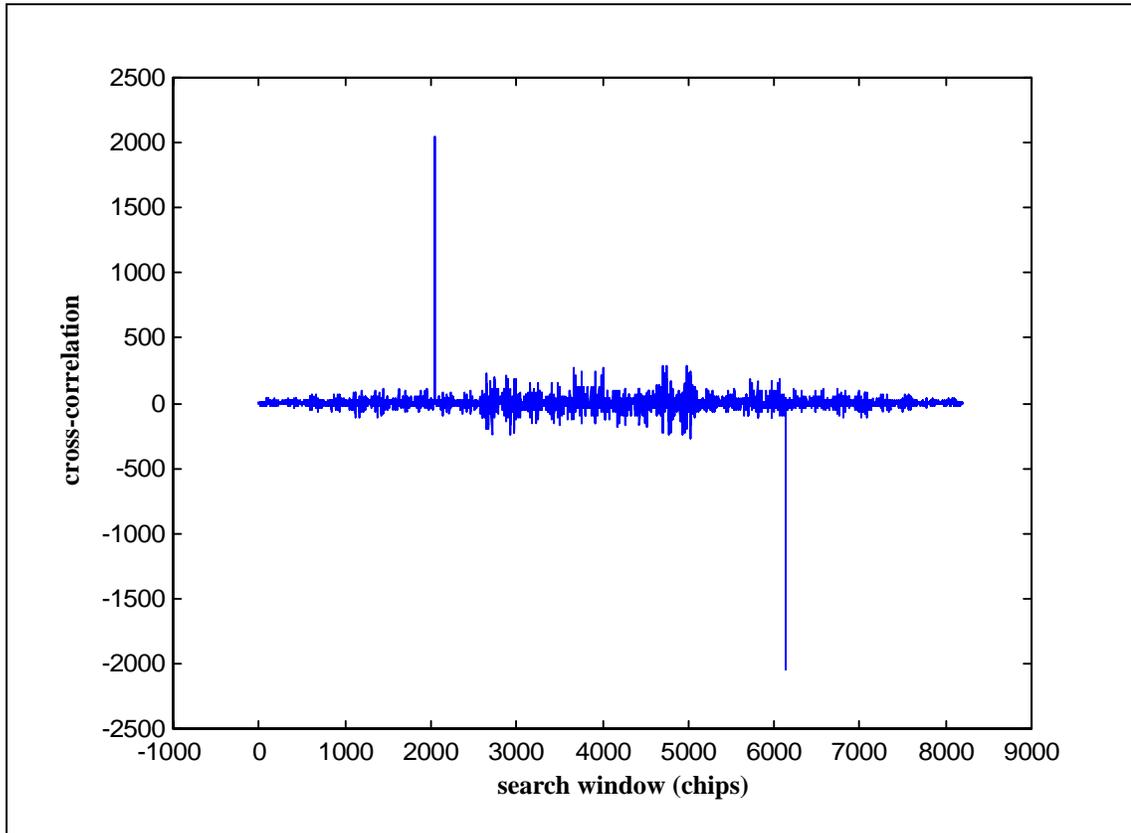
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### 1.2.1 Performance Issues

However, both Gold and Golay based sequence possesses an inherent draw back that such a concatenated preamble will have undesirable high energy of cross-correlation among the signatures at multiple time offset of 256 chips. This implies that the current RACH preamble and signature can not work satisfactorily in the cell radius larger than 9 kms.

This constitutes a fundamental challenge of concatenation of preamble short scrambling sequence and signature for large searching window applications. In other words, the excessive large cross-correlation peaks between the signatures will always exist if the search window is larger than the length of scrambling sequence.

**FIGURE 1. Cross-Correlation Between Signature #1 and #6 in Table 1**



### 1.3 Nokia Proposal for Long PN Scrambling Sequence Based RACH Preamble

The Nokia proposal is based on the idea of using long PN code as preamble sequence of length 4096, then such a preamble is concatenated by a signature with 16 symbols, there are 16 signatures in total. In this arrangement, we can use the same match filter structure to match 256 chips segmented PN sequence at first stage and then to correlate the 16 signatures set to detect the preamble. The classical orthogonal Gold sequence can be used as the signature. This scheme can completely eliminate the cross-correlation peaks at the

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uncertainty delay of multiple 256chips instants. However, the superior aperiodic auto-correlation property of Golay sequence and its associated very efficient hardware implementation (so-called EGC) advantages can not be retained.

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## 2.0 Interleave Concatenation of Golay Sequence

### 2.1 Generation of Golay Sequence by Transformations

In this contribution, we present a simple construction technique which allows to generate the 4096 long Golay sequence based on the 256 constituent Golay sequence. The preamble signature can be constructed by interleaving concatenation of the constituent Golay sequence A,B. In order to keep the EGC hardware simple, we propose several transforms which can generate additional Golay sequence based on the constituent Golay sequence A and B. As such we can construct longer Golay sequence which allow the elimination of the high cross-correlation spikes for the multiple delay of 256 chips.

We propose to use the following 3 transformations to construct new Golay sequence based on constituent sequence A,B only.

#### 1. Sequence Reversal

We defined a new Golay sequence by reverse ordering the sequence of constituent sequence A,B:

$$\text{For } \mathbf{A} = [a_1, a_2, \dots, a_{N-1}, a_N]; \mathbf{B} = [b_1, b_2, \dots, b_{N-1}, b_N]$$

$$\text{then } \bar{\mathbf{A}} = [a_N, a_{N-1}, \dots, a_2, a_1]; \bar{\mathbf{B}} = [b_N, b_{N-1}, \dots, b_2, b_1]$$

It can be shown pair  $\bar{\mathbf{A}}, \bar{\mathbf{B}}$  is a Golay pair sequence.

#### 2. Address Bit Reversal

We defined a new Golay sequence by address bit reverse ordering the sequence of constituent sequence A,B:

For  $\mathbf{A} = [a_1, a_2, \dots, a_{N-1}, a_N]; \mathbf{B} = [b_1, b_2, \dots, b_{N-1}, b_N]$ . the index can be represented in binary form (8 bits for L=256). i.e.  $(1)_{decimal} = (00000001)_{binary}$ . In this case, the so-called address bit reversal is defined as  $Rev(1)_{decimal} = Rev(00000001)_{binary}$ ,  $= (10000000)_{binary} = (128)_{decimal}$ , in fact, this is a random permutation of the position of constituent Golay sequence. Therefore we have a new pair of Golay sequence:

$$\mathbf{A}^r = [a_{rev(1)}, a_{rev(2)}, \dots, a_{rev(N-1)}, a_{rev(N)}];$$

$$\mathbf{B}^r = [b_{rev(1)}, b_{rev(2)}, \dots, b_{rev(N-1)}, b_{rev(N)}];$$

It can be shown pair  $\mathbf{A}^r, \mathbf{B}^r$  is also a Golay pair sequence.

### 3. Resampling

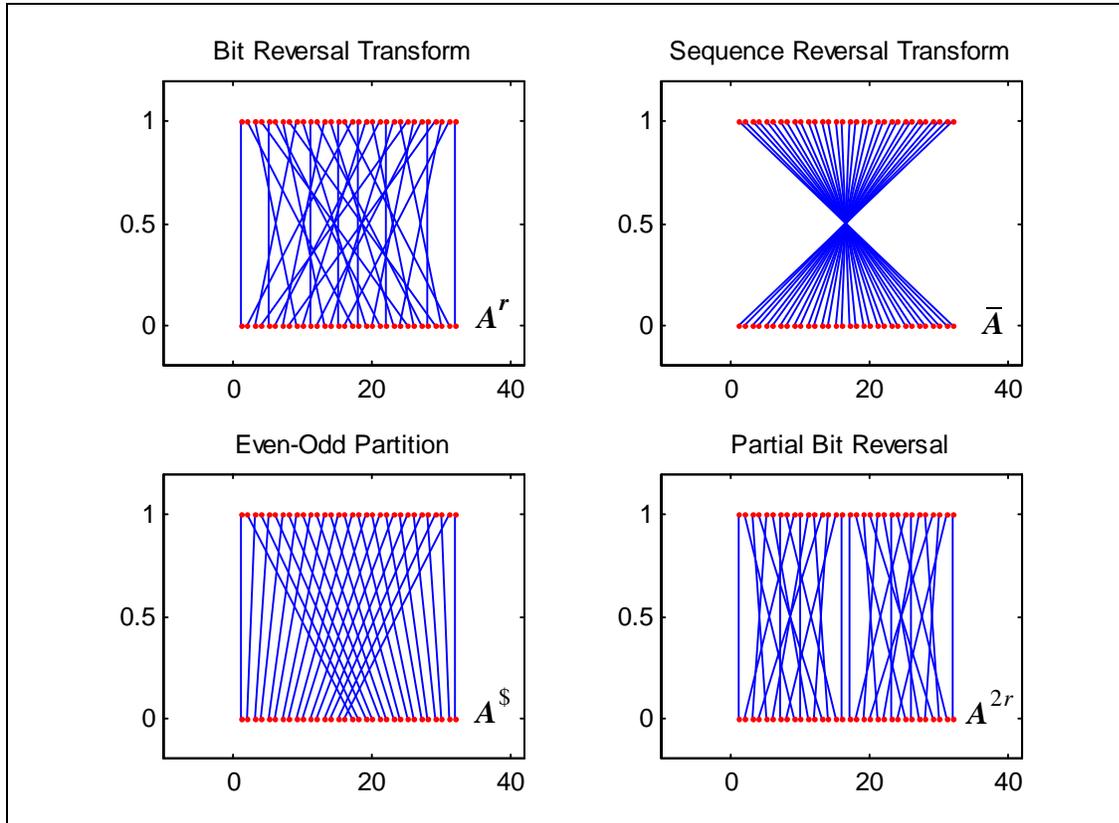
We defined a new Golay sequence by concatenation of two sub-sequence obtained via decimate sampling.

$$\text{For } \mathbf{A} = [a_1, a_2, \dots, a_{N-1}, a_N]; \mathbf{B} = [b_1, b_2, \dots, b_{N-1}, b_N];$$

$$\text{then } \mathbf{A}^{\$} = [a_1, a_3, \dots, a_{N-1}, a_2, a_4, \dots, a_N]; \mathbf{B}^{\$} = [b_1, b_3, \dots, b_{N-1}, b_2, b_4, \dots, b_N];$$

It can be shown pair  $\mathbf{A}^{\$}, \mathbf{B}^{\$}$  is a Golay pair sequence.

**FIGURE 2. Examples of Transform for a 32 chips Sequence**



## 2.2 Extended Long Golay Sequence and Hadamard Signature

Using single pair of constituent Golay sequence  $A, B$ , by transformation, we can generate long Golay sequence. The key issues here is how to construct the signature to achieve superior aperiodic auto-cross correlation properties. In the following we presented the following 3 approaches. In Table 2, we propose one variant of extended Golay sequence by transformation, based on the transform depicted in Figure 2. Table 3 lists the signatures based on the Golay sequence  $A, B, C, D, E, F, G, H, A1, B1, C1, D1, E1, F1, G1, H1$ .

TABLE 2. Definition of Extended Golay Sequence

|  | A | B | C         | D         | E     | F     | G           | H           | A1    | B1    | C1          | D1          | E1       | F1       | G1             | H1             |
|--|---|---|-----------|-----------|-------|-------|-------------|-------------|-------|-------|-------------|-------------|----------|----------|----------------|----------------|
|  | A | B | $\bar{A}$ | $\bar{B}$ | $A^r$ | $B^r$ | $\bar{A}^r$ | $\bar{B}^r$ | $A^s$ | $B^s$ | $\bar{A}^s$ | $\bar{B}^s$ | $A^{2r}$ | $B^{2r}$ | $\bar{A}^{2r}$ | $\bar{B}^{2r}$ |

TABLE 3. Hadamard-Walsh Signature

| Signature | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  |
|-----------|---|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1         | A | B  | C  | D  | E  | F  | G  | H  | A1  | B1  | C1  | D1  | E1  | F1  | G1  | H1  |
| 2         | A | -B | C  | -D | E  | -F | G  | -H | A1  | -B1 | C1  | -D1 | E1  | -F1 | G1  | -H1 |
| 3         | A | B  | -C | -D | E  | F  | -G | -H | A1  | B1  | -C1 | -D1 | E1  | F1  | -G1 | -H1 |
| 4         | A | -B | -C | D  | E  | -F | -G | H  | A1  | -B1 | -C1 | D1  | E1  | -F1 | -G1 | H1  |
| 5         | A | B  | C  | D  | -E | -F | -G | -H | A1  | B1  | C1  | D1  | -E1 | -F1 | -G1 | -H1 |
| 6         | A | -B | C  | -D | -E | F  | -G | H  | A1  | -B1 | C1  | -D1 | -E1 | F1  | -G1 | H1  |
| 7         | A | B  | -C | -D | -E | -F | G  | H  | A1  | B1  | -C1 | -D1 | -E1 | -F1 | G1  | H1  |
| 8         | A | -B | -C | D  | -E | F  | G  | -H | A1  | -B1 | -C1 | D1  | -E1 | F1  | G1  | -H1 |
| 9         | A | B  | C  | D  | E  | F  | G  | H  | -A1 | -B1 | -C1 | -D1 | -E1 | -F1 | -G1 | -H1 |
| 10        | A | -B | C  | -D | E  | -F | G  | -H | -A1 | B1  | -C1 | D1  | -E1 | F1  | -G1 | H1  |
| 11        | A | B  | -C | -D | E  | F  | -G | -H | -A1 | -B1 | C1  | D1  | -E1 | -F1 | G1  | H1  |
| 12        | A | -B | -C | D  | E  | -F | -G | H  | -A1 | B1  | C1  | -D1 | -E1 | F1  | G1  | -H1 |
| 13        | A | B  | C  | D  | -E | -F | -G | -H | -A1 | -B1 | -C1 | -D1 | E1  | F1  | G1  | H1  |
| 14        | A | -B | C  | -D | -E | F  | -G | H  | -A1 | B1  | -C1 | D1  | E1  | -F1 | G1  | -H1 |
| 15        | A | B  | -C | -D | -E | -F | G  | H  | -A1 | -B1 | C1  | D1  | E1  | F1  | -G1 | -H1 |
| 16        | A | -B | -C | D  | -E | F  | G  | -H | -A1 | B1  | C1  | -D1 | E1  | -F1 | -G1 | H1  |

Note that the 4 transformation can be implemented every simple way, namely by reconfigure the data bus connection. Therefore the transform operations add no implementation complexity.

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## 2.3 Properties of the Proposed RACH Preamble

The proposed RACH preamble can keep the good correlation properties unique to Golay sequence. Instead of repeating the two constituent Golay pair sequence A,B, we use simple permutation transform to extend the two constituent Golay pair sequence to 8 pairs. Based on this framework, we have the following advantages:

1. Completely eliminate the high energy cross-correlation peaks.
2. Preamble optimization can be done by selecting the best combinations of the transformation, also by selecting the best  $P$  ( in Ref.[1] ) vector value required to generate constituent Golay pair A,B.
3. The extended Golay sequence do not require a special designed signature matrix as proposed by Ericsson in Table 1, nor it is necessary to use orthogonal Gold sequence to achieve good performance, it lends itself a direct concatenation with Hadamard-Walsh matrix, which allows a well known FHT implementation of signature detector.
4. By permuting the order of the extended Golay sequence, e.g.in Table 2, we can enlarge the signature set more than 32 as suggested by Ericsson's proposal.

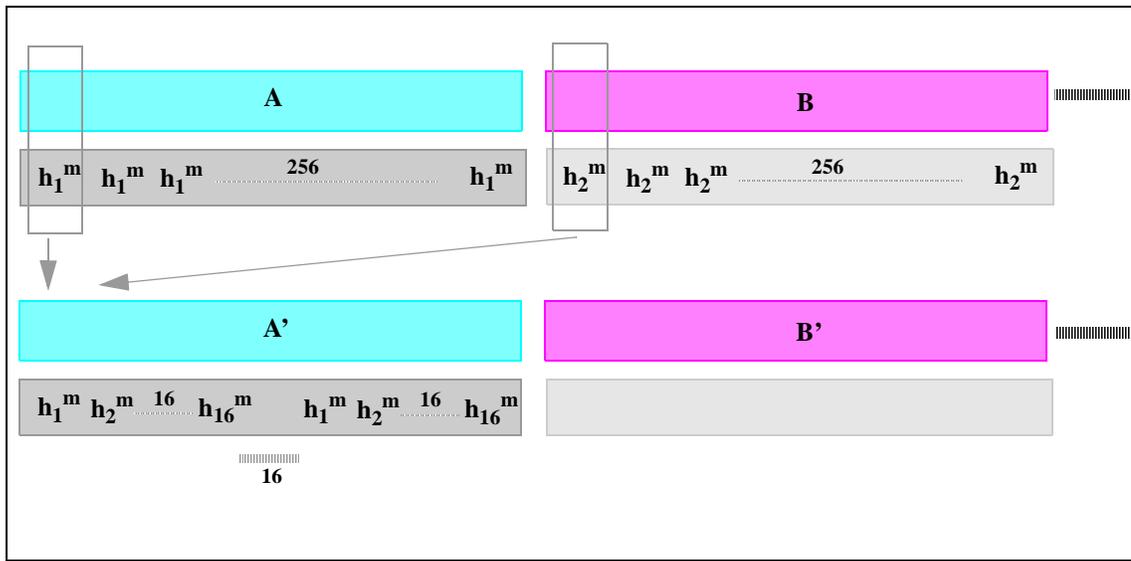
The efficient implementation of RACH preamble and signature will be addressed in a separate contribution.

## 2.4 Doppler Resistant RACH Preamble Construction

In a very recent work from Motorola and TI Ref [4], a new concatenation of the preamble scrambling sequence and signature based on different mapping is proposed, it is shown that such a novel construction can reduce the cross-correlation in the presence of high Doppler shift or initial frequency offset. In what follows, we show that such an idea can be extended to the proposed the RACH preamble signature in a straightforward fashion.

Consider the 16 preambles defined in Table 2 &3 and they are Golay sequence. Now we shall expand the re-sampling transformation defined Section 2.1, namely the even-odd partition transformation in Figure 2. In Figure 2, we use only 2-subset partition, if we apply 16 sub-sets partition, we can obtain the same permeable signature as proposed by Motorola and TI.

**FIGURE 3. Generation of Doppler Resistant RACH Preamble**



The detailed procedure is shown in Figure 3.

As we can show, such a Golay sequence based Doppler resistant preamble can achieve the same performance as Long PN sequence based Doppler resistant preamble proposed by Motorola and TI.

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## 3.0 Performance Evaluation

### 3.1 Inter-Signature Aperiodic Cross Correlation

In this section, we present a performance evaluation results for the RACH preambles proposed by Ericsson, Nokia, Motorola/TI and Nortel. This evaluation is based on the a-periodic auto-cross correlation properties of the different proposed preamble and signatures, we exam all the 136 paris inter-signature auto/cross correlations, with respect to different search window size.

In Figure 4, we can see Ericsson preamble suffers from inter-signature cross-correlation peak. The Nortel preamble can eliminate such undesirable peaks.

FIGURE 4. Correlation of Ericsson//Nortel Preambles

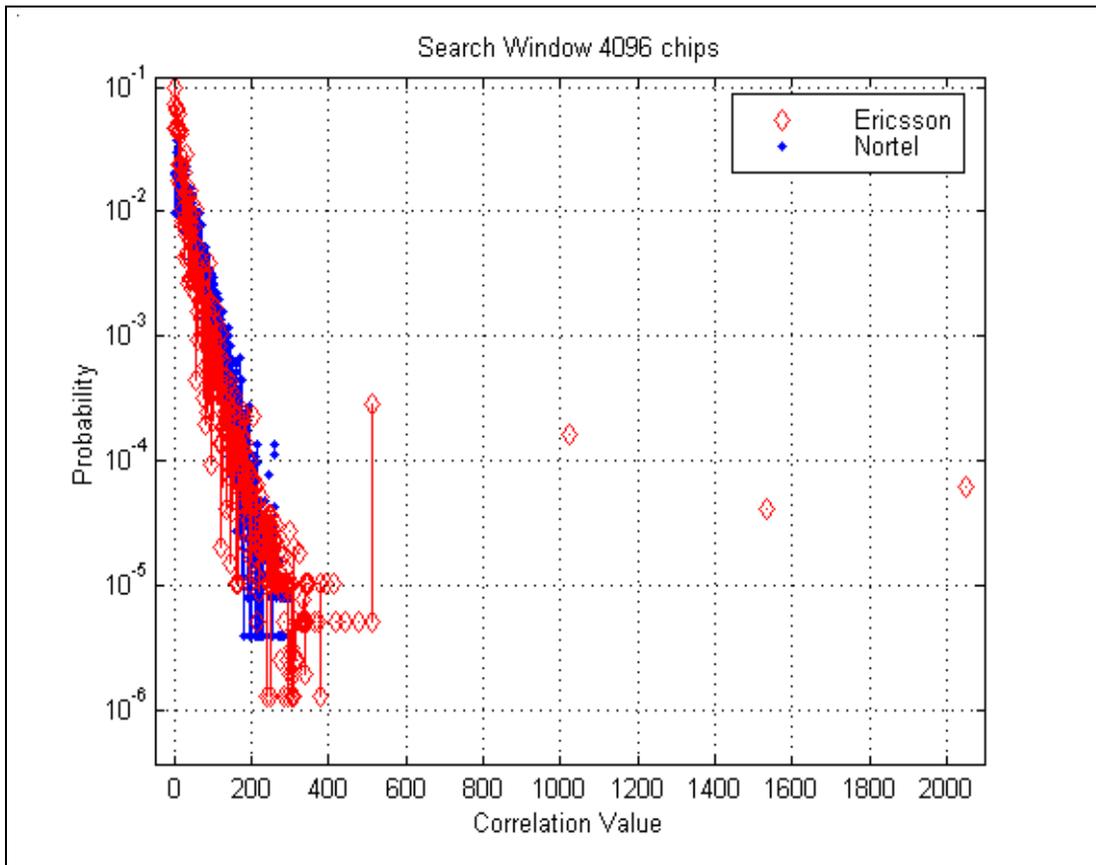
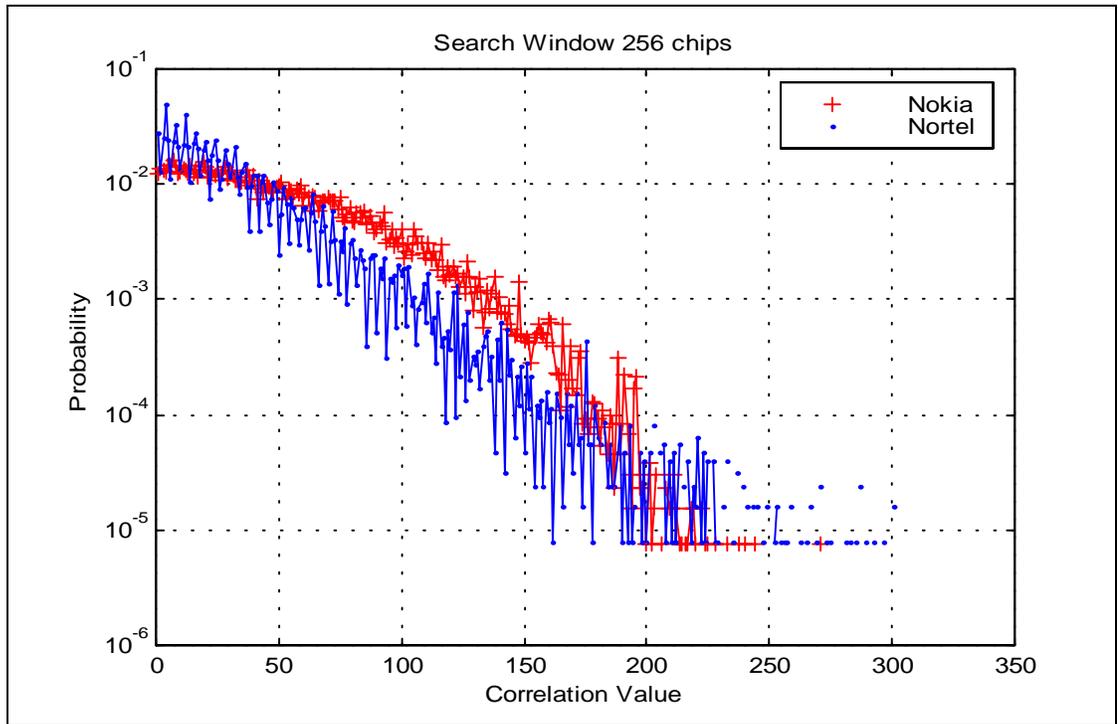
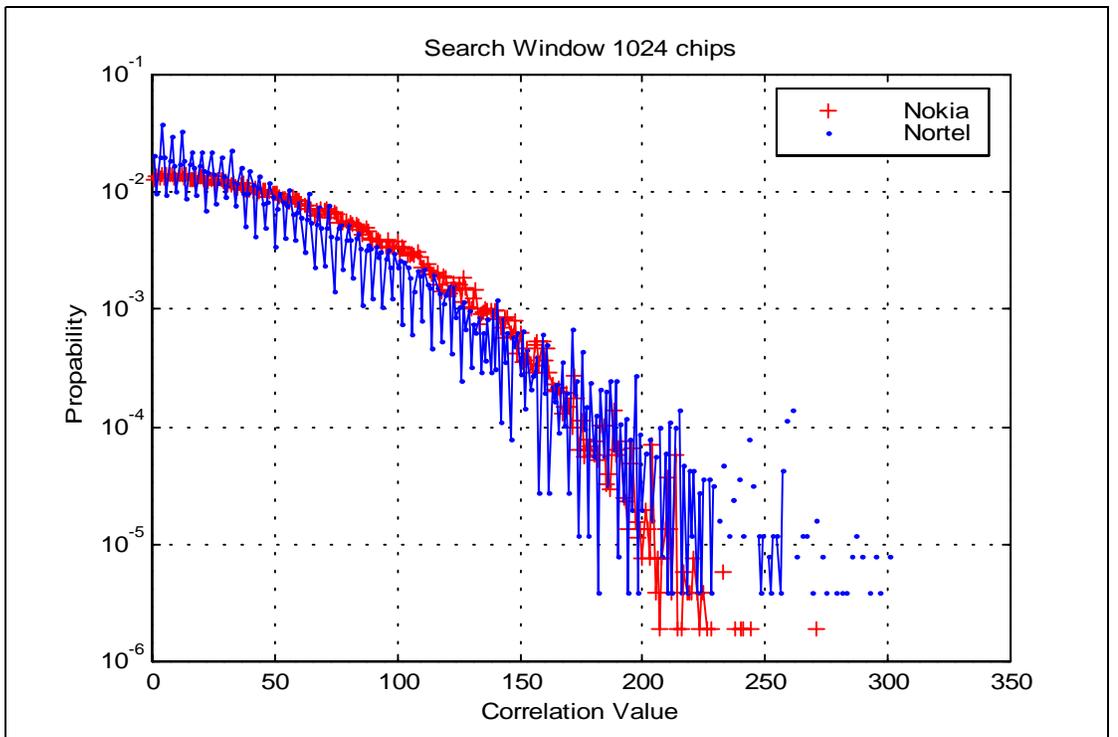


Figure 5 -7 depict the distribution of the inter-signature correlation for Nokia long code based and Nortel Golay based preambles based on the different search window. As we can see the long code based preamble exhibit the almost the same with respect to different search window size, while Nortel Golay sequence based preamble has advantage for typical search window size. This can be demonstrated in Figure 8-10.

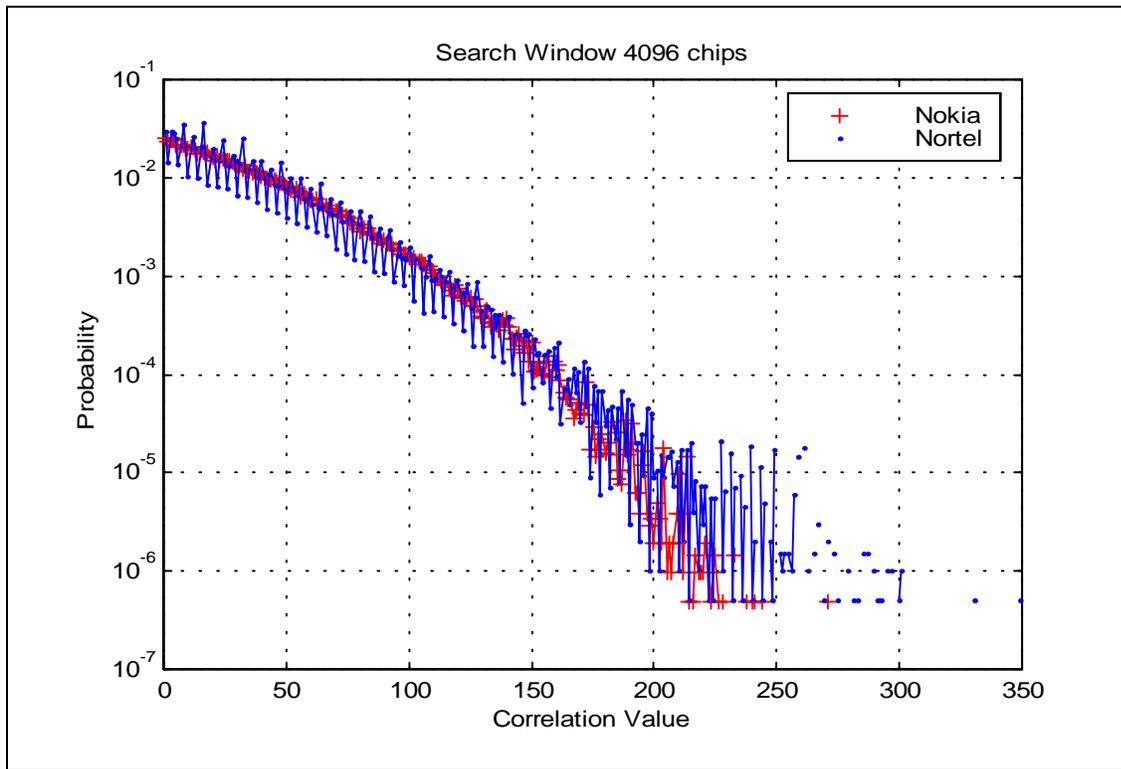
**FIGURE 5. Inter-Signature Correlation for Nokia and Nortel Preambles (search window=256)**



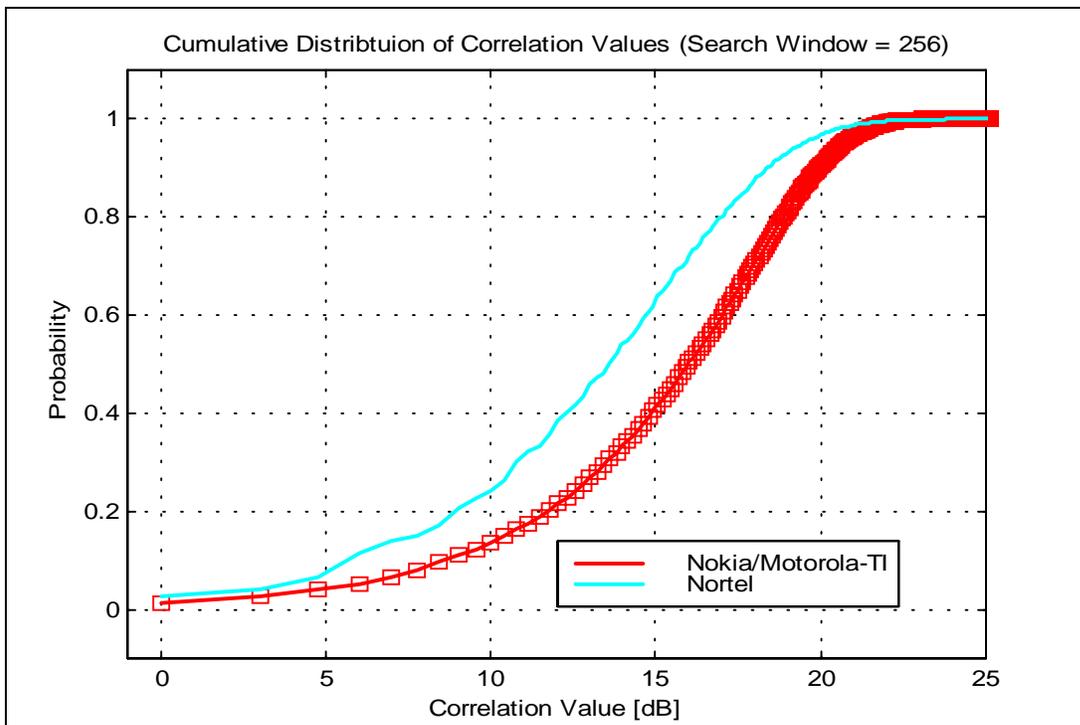
**FIGURE 6. Inter-Signature Correlation for Nokia and Nortel Preambles (search window=1024)**



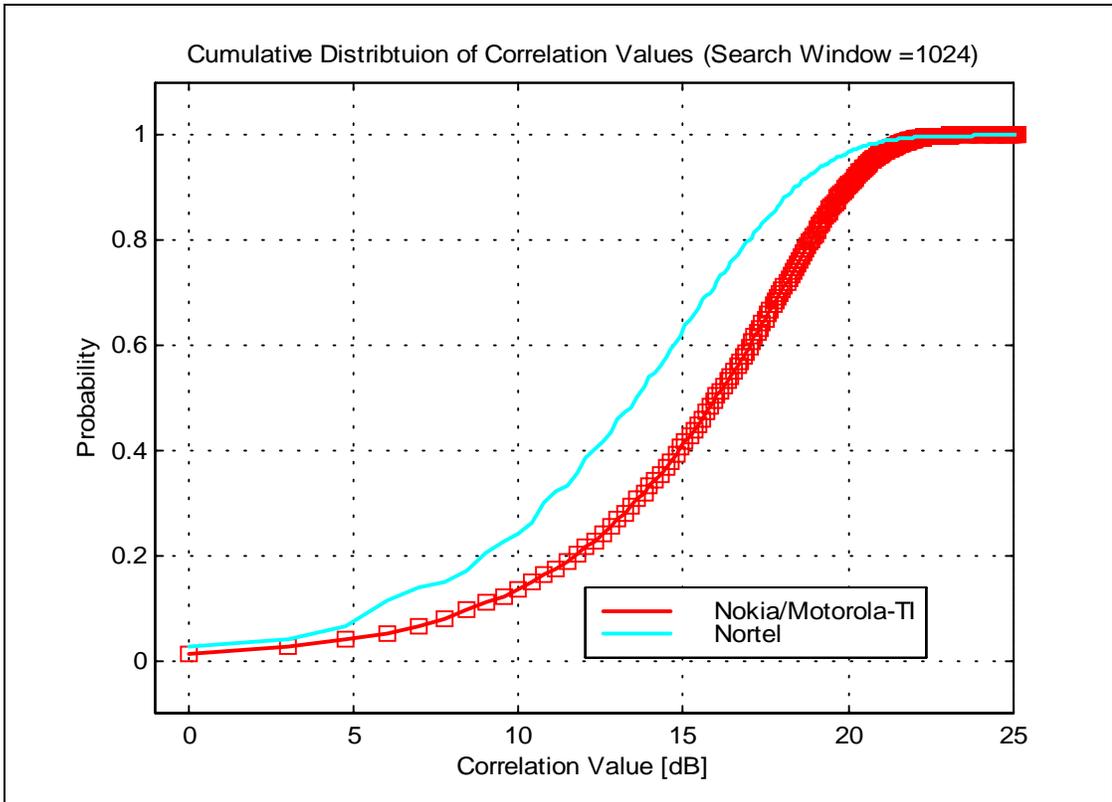
**FIGURE 7. Inter-Signature Correlation for Nokia and Nortel Preambles (search window=4096)**



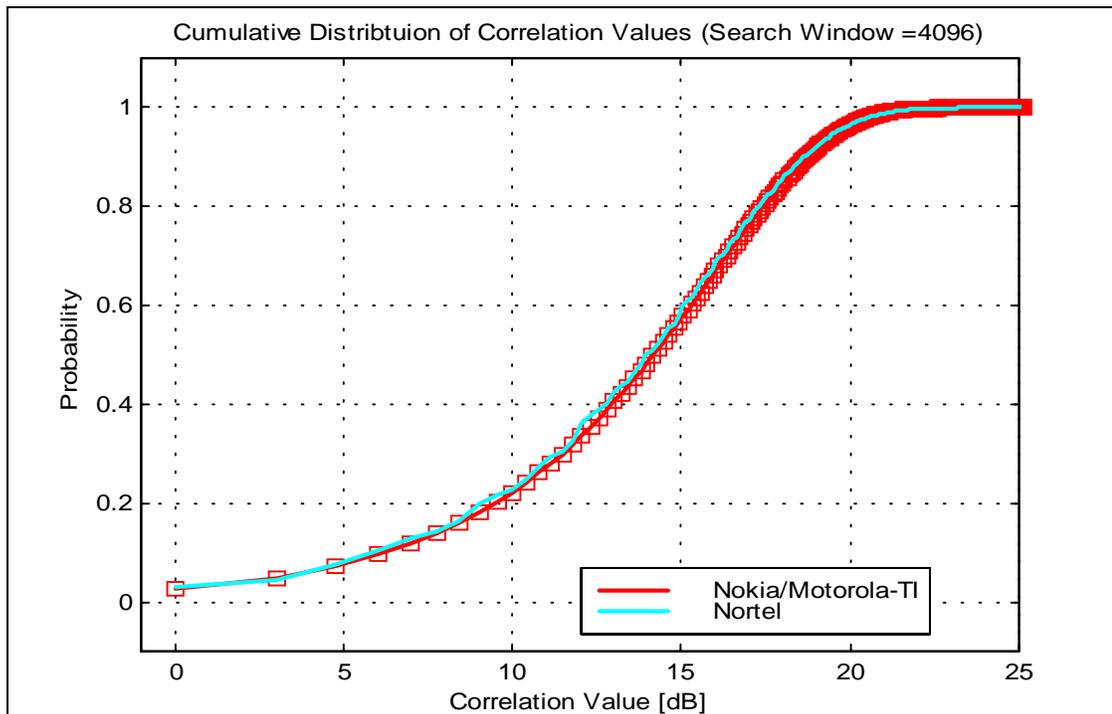
**FIGURE 8. Inter-Signature Correlation CDF of Nokia and Nortel Preamble (search window=256)**



**FIGURE 9. Inter-Signature Correlation CDF of Nokia and Nortel Preamble (search window=1024)**



**FIGURE 10. Inter-Signature Correlation CDF of Nokia and Nortel Preamble (search window=4096)**



As we can see for the typical cell size deployment, Golay preamble possesses lower inter-signature correlation than the Long Code PN preamble.

### 3.2 Aperiodic Auto-Correlation Evaluation

Figure 11 shows the CDF of aperiodic auto-correlation for all signatures for both Golay preamble and Long PN code preamble. In this case the searching window is 256 chips. It can be seen that the Golay preamble possesses superior auto-correlation than Long PN code preamble. Due to fact that the Hadamard matrix is used as signature matrix, the auto-cross correlation properties of concatenated Golay sequence is also determined by correlation property of Hadamard matrix. On the other hand, given the Golay sequence, the signature can be further optimized, however, the signature detector can be implemented as FHT. In general, using the Hadamard signature, the superior aperiodic auto-correlation and inter-signature correlation of the Golay preamble is significantly better. See Figure 8,9,11,12 with the search window of 1024 chips.

FIGURE 11. Aperiodic auto-correlation for all 16 Signatures (search window=256)

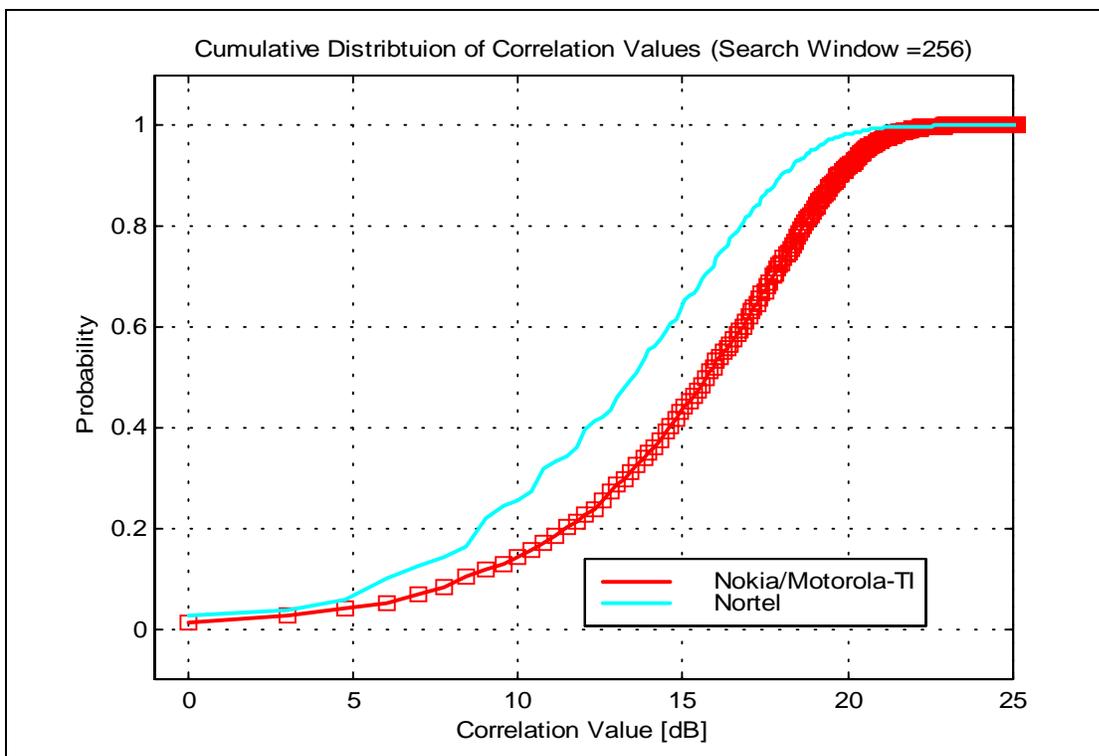
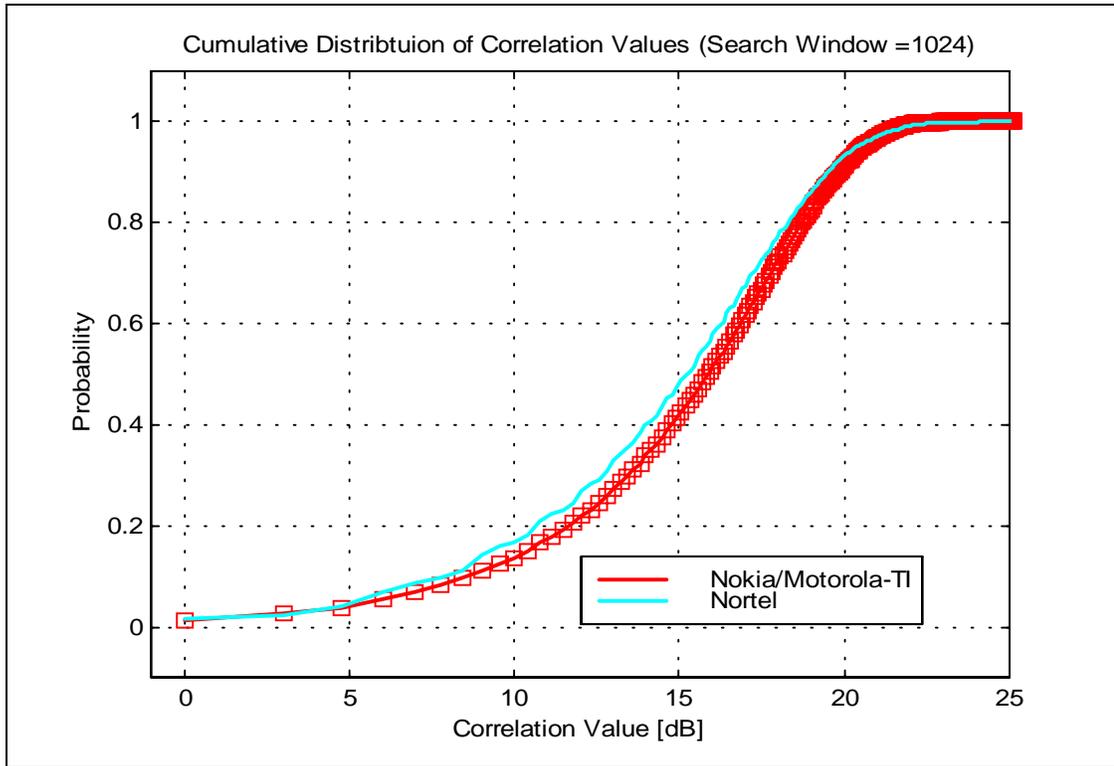


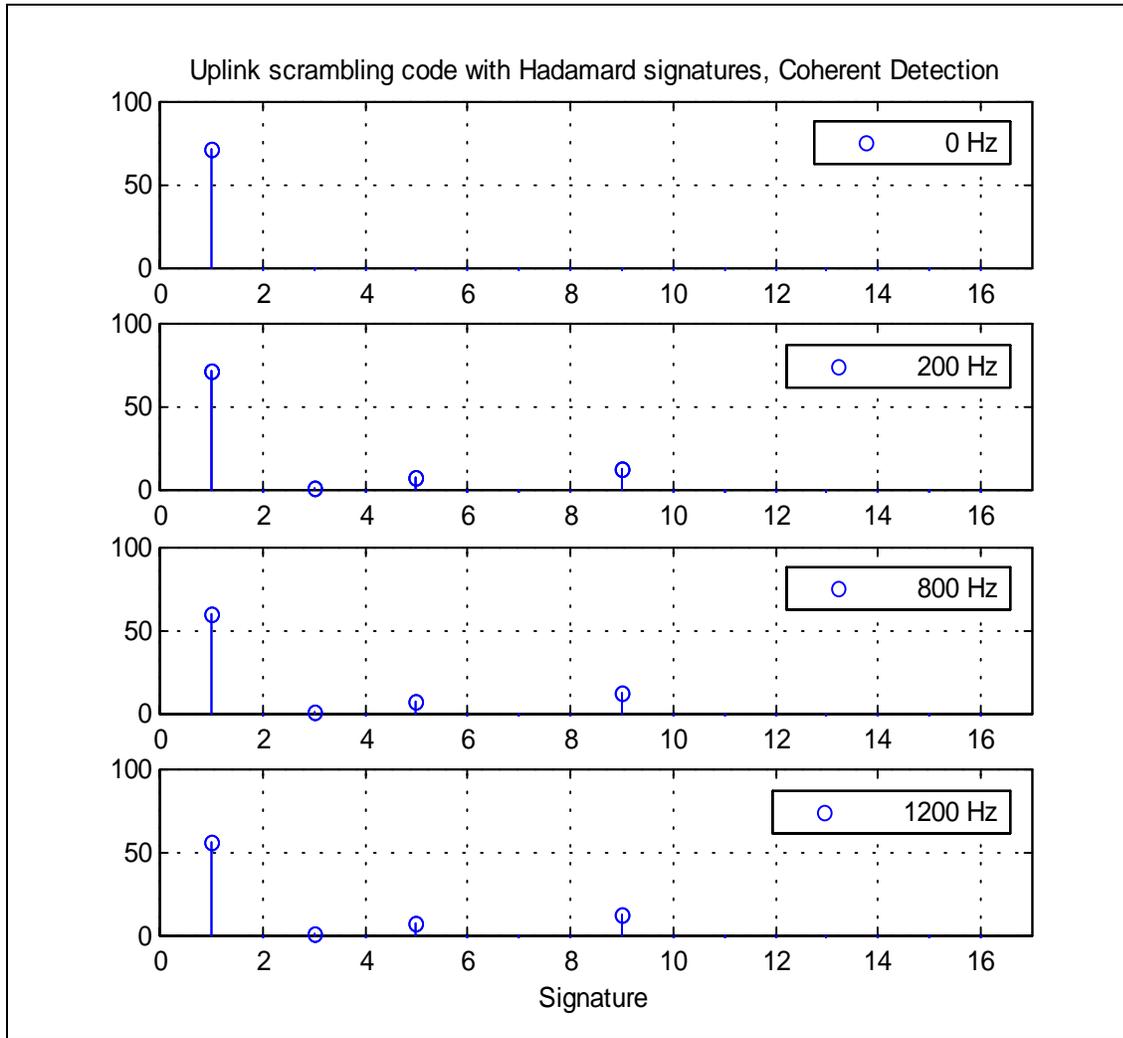
FIGURE 12. Aperiodic Atuo-correlation for all 16 Signature (search window=1024)



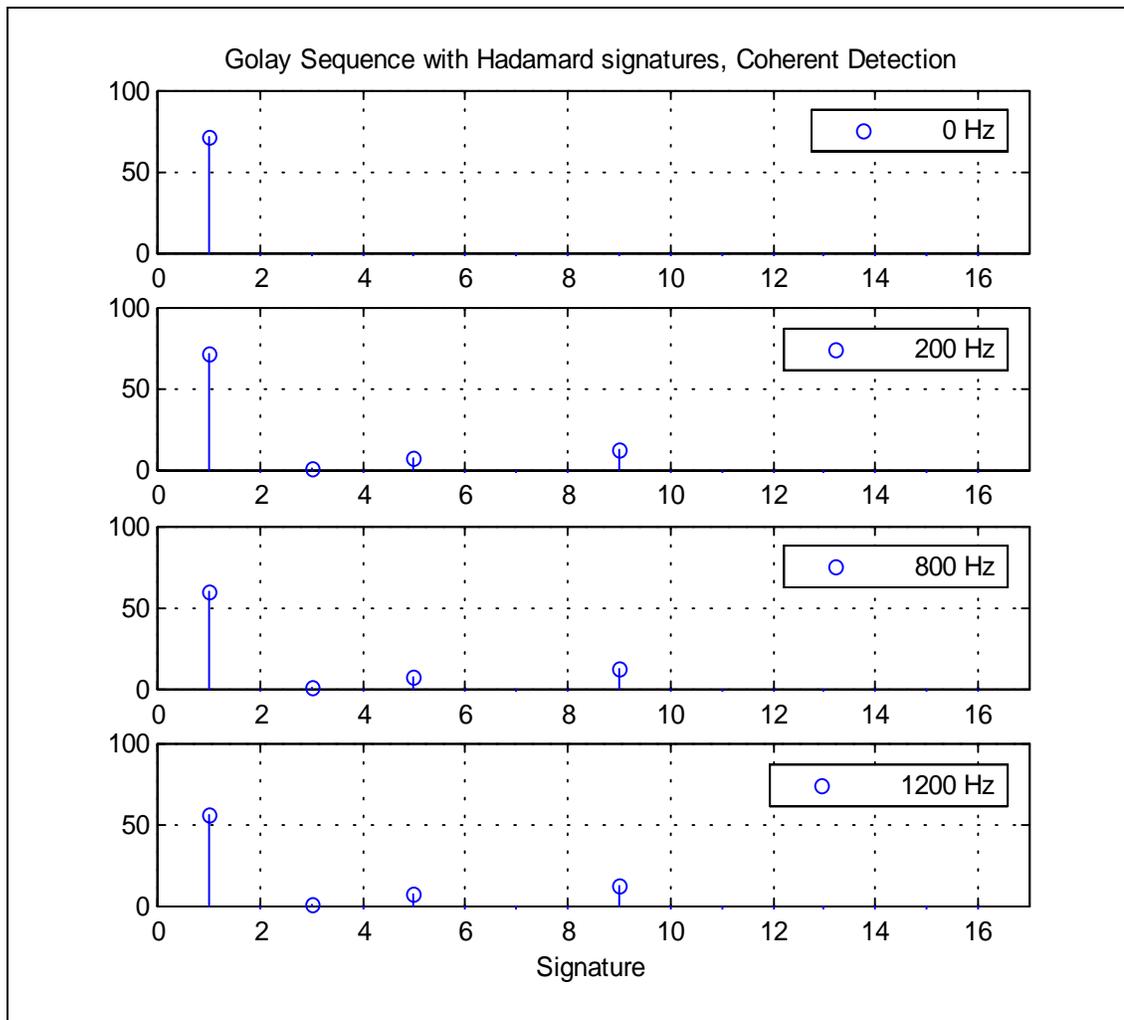
### 3.3 Doppler Robustness Evaluation

Figure 13, show the Motorola/TI long code based preamble inter-signature correlation performance in the presence of high frequency offset or Doppler shift. Figure 14 is the Nortel Golay preamble based on the procedure in Section 2.4 and Figure 3. As we can see both preambles have the same robustness performance in high Doppler detection environment.

**FIGURE 13. Motorola/TI Preamble Inter-Signature Correlation in the Presence of High Doppler Shift**



**FIGURE 14. Nortel Preamble Inter-Signature Correlation in the Presence of High Doppler Shift**



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## 4.0 Summary and Conclusions

In this contribution, we extended the 256 short Golay sequence to 4096 chips long Golay sequence as RACH preamble scrambling code. Such a construction is based on the simple transform of constituent Golay pair sequence A,B. The extended Golay sequence allows direct use Hadamard matrix as signature. The proposed Golay-Hadamard RACH preamble possesses superior aperiodic auto-correlation and inter-signature correlation properties than the Long PN code based preamble, especially, when the search window is operating within 1024 chips. In addition, the proposed Golay-Hadamard preamble can be directly transform to the Doppler resistant preamble proposed by Motorola and TI.

### References:

- [1] TSGR1#3(99)205: *New RACH Preambles with Low Auto-Correlation Sidelobes and Reduced Detector Complexity*. March, 1999, Source: Ericsson
- [2] TSGR1#5(99)599: *Further Clarification of Nokia's RACH Preamble Proposal*. June, 1999, Source: Nokia
- [3] TSGR1#5(99)670: *Evaluation of Proposed RACH Signatures*, June, 1999, Source: Motorola
- [4] TSGR1#6(99)893: *Proposal for RACH Preambles*, July, 1999, Source: Motorola and Texas Instruments

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## **0.0 Summary**

In this contribution, we present an interleave concatenation of Golay and Hadamard sequence to construct the RACH preamble and signatures for 3GPP in large cell operation.

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## 5.0 Implementation of Golay Correlator

### 5.1 Recursive Golay Sequence Generator

Golay sequence can be constructed by the following generalized algorithm:

$$\begin{aligned}
 a_0(i) &= \delta(i) \\
 b_0(i) &= \delta(i) \\
 a_{n+1}(i) &= a_n(i) + W_n b_n(i - 2^{P_n}) \\
 b_{n+1}(i) &= a_n(i) - W_n b_n(i - 2^{P_n})
 \end{aligned}$$

where  $0 < i < 2^n - 1$  otherwise  $a_n(i) = b_n(i) = 0$ .  $W_n = \pm 1$  and vector  $P_n$  determines the generation of different Golay sequence.

### 5.2 Lattice Structure Golay Correlator

By recursion definition, we can show that the complementary pair of Golay sequence A and B have the following auto-correlation expressions:

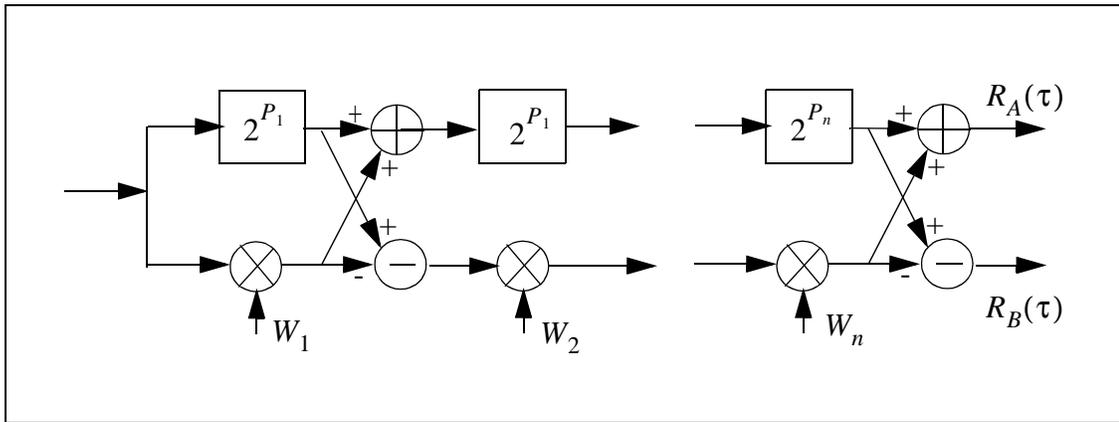
$$\begin{aligned}
 R_{n+1}^a(k) &= \sum \{a_{n+1}(i)a_{n+1}^*(i+k)\} \\
 &= \sum \left\{ a_n(i) + W_n b_n(i - 2^{P_n}) \right\} \left\{ a_n^*(i+k) + W_n b_n^*(i+k - 2^{P_n}) \right\} \\
 &= R_n^a(k) + |W_n|^2 R_n^b(k) + W_n a_n^*(i+k) b_n(i - 2^{P_n}) \\
 &\quad + W_n^* a_n(i) b_n^*(i+k - 2^{P_n})
 \end{aligned}$$

and for sequence B we have,

$$\begin{aligned}
 R_{n+1}^b(k) &= \sum \{b_{n+1}(i)b_{n+1}^*(i+k)\} \\
 &= \sum \left\{ a_n(i) + W_n b_n(i - 2^{P_n}) \right\} \left\{ a_n^*(i+k) - W_n b_n^*(i+k - 2^{P_n}) \right\} \\
 &= R_n^a(k) + |W_n|^2 R_n^b(k) + W_n a_n^*(i+k) b_n(i - 2^{P_n}) \\
 &\quad - W_n^* a_n(i) b_n^*(i+k - 2^{P_n})
 \end{aligned}$$

This immediately leads to a lattice implementation of the Efficient Golay Correlator (EGC). see Figure 2.

**FIGURE 15. Efficient Golay Correlator**



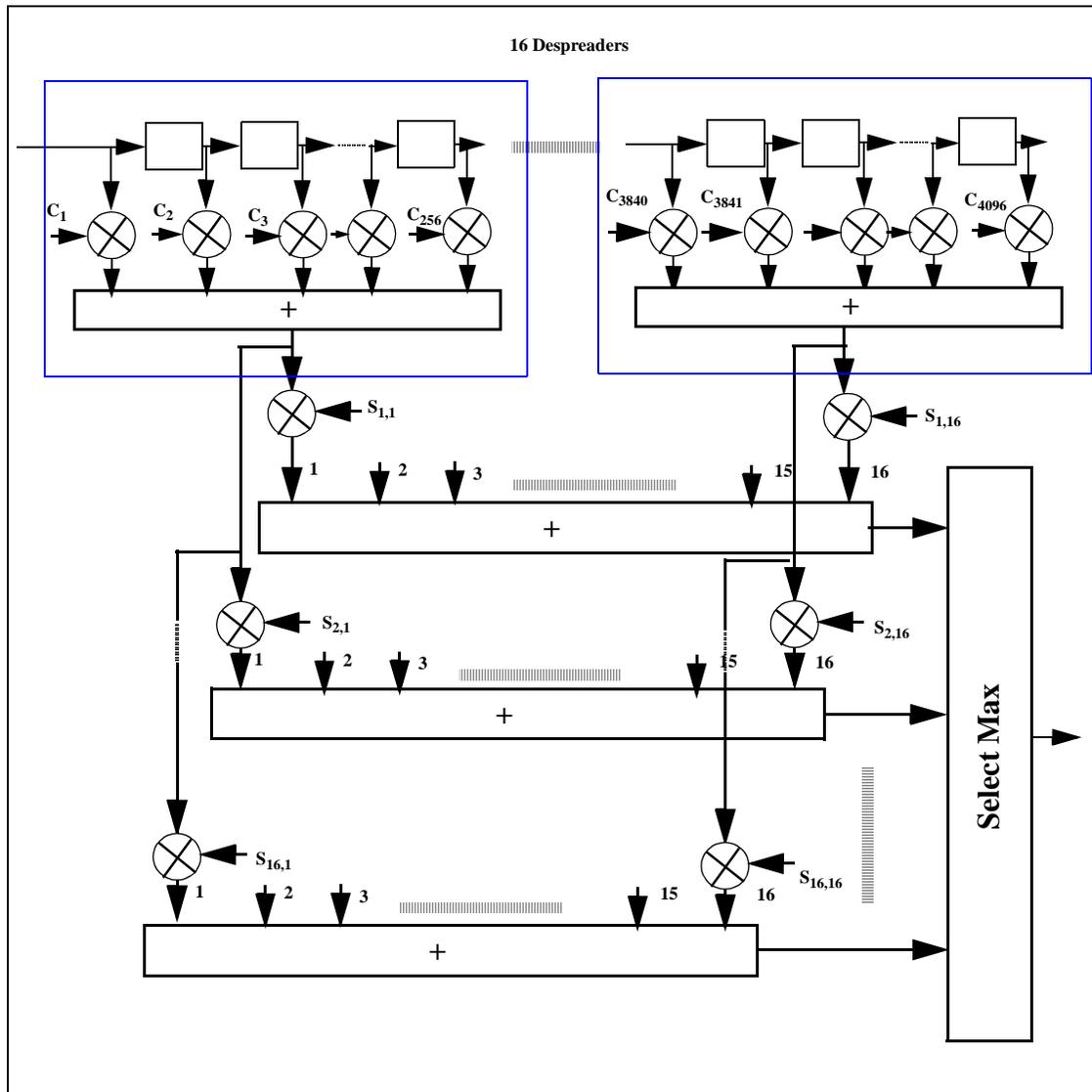
The memory requirement is  $L$  samples ( $L=256$ ). The adder/subtractor requirement is  $.2 \times \log L = 16$ . Such a lattice structure correlator is well known. e.g. Ref[2].

Based on the Golay and Hadamard concatenation preamble, the preamble sequence is constructed based on the permutation of the constituent Golay sequence A,B. Therefore on the efficient Golay correlator structure in Figure 3, we should base on the order the permutation to de-interleave the permuted input samples and feed to EGC to correlate with A,B. In this case the add on complexity is only the address generation part. In particular, since we use Hadamard sequence as signature, the 16 signature correlator bank can be reduced to the FHT.

## 6.0 Implementation of RACH Preamble Detector

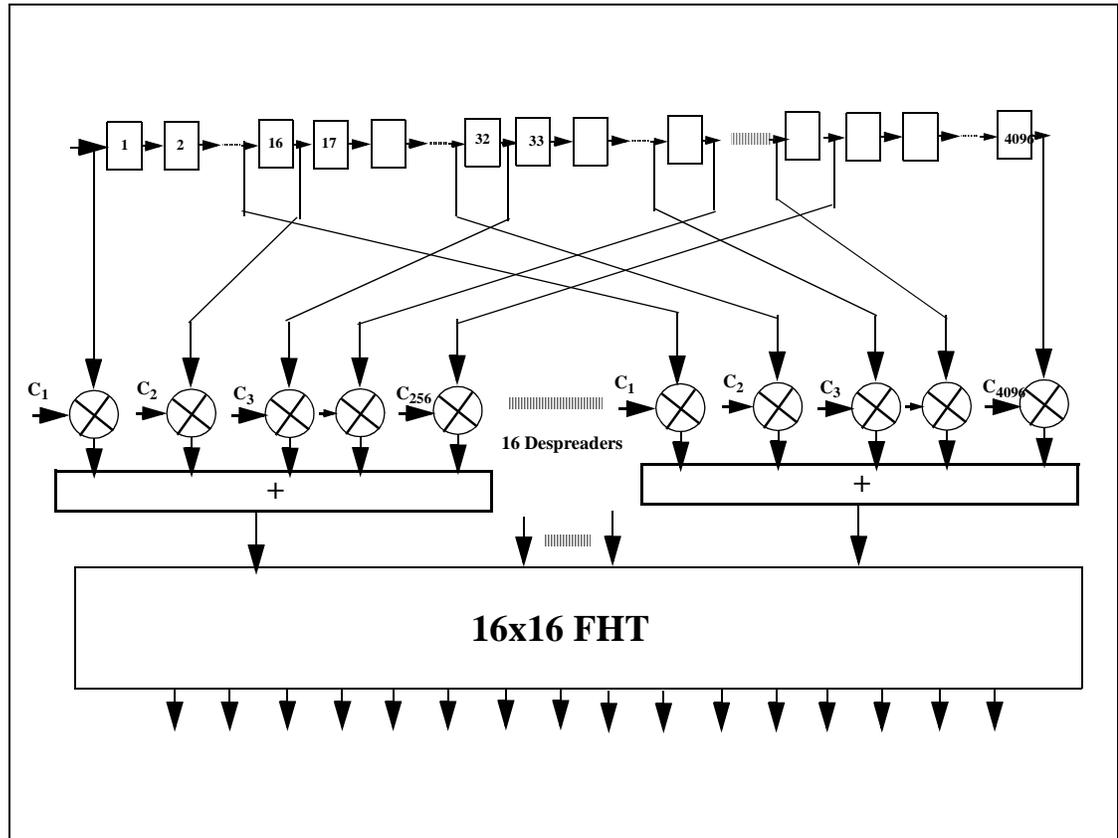
### 6.1 Despreader Based Match Filter

FIGURE 16. Nokia RACH Preamble Detector Architecture



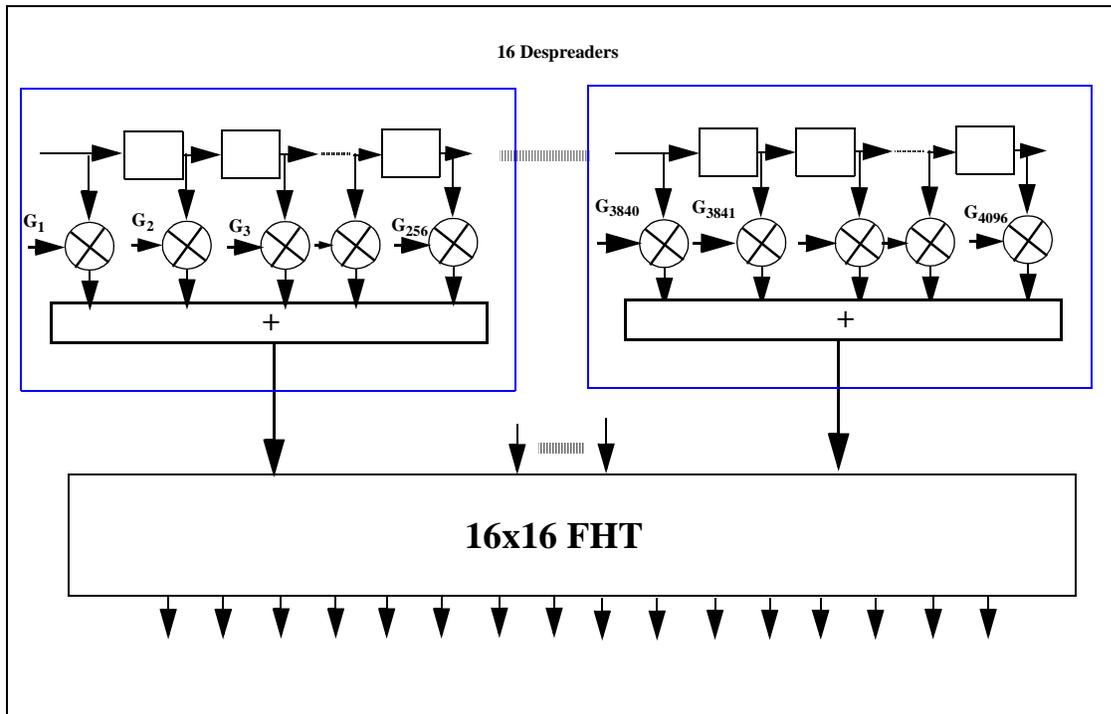
## 6.2 Doppler Resistant RACH Preamble Detector

FIGURE 17. Motorola/TI Preamble Detector Architecture



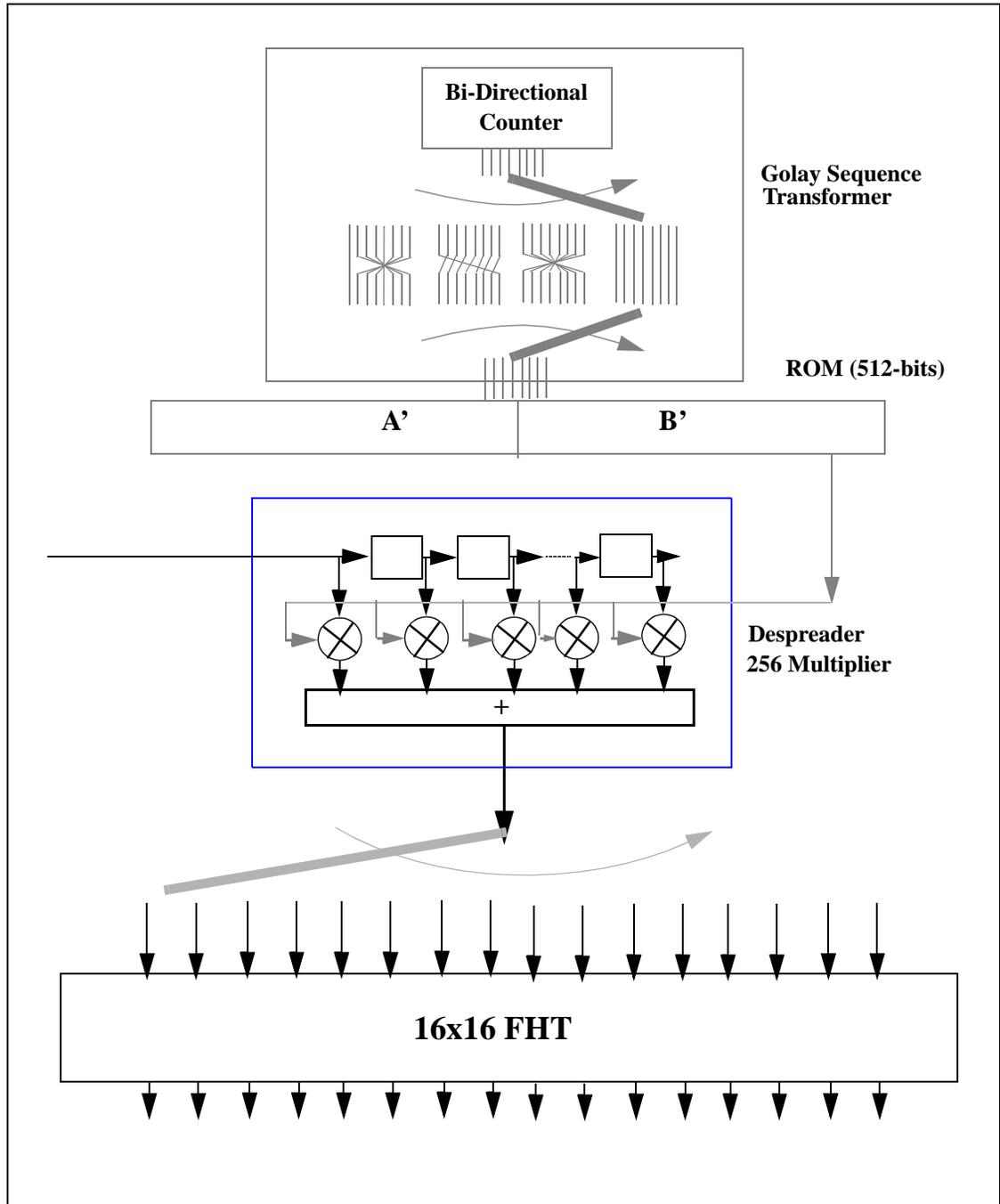
### 6.3 Despreader Based Golay Hadamard Preamble Detector

FIGURE 18. Nortel Golay-Hadamard Preamble Detector Based on Nokia Architecture



## 6.4 Efficient Golay Hadamard Preamble Detector

FIGURE 19. Efficient Nortel Golay-Hadamard Preamble Detector (Architecture-I)



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## 7.0 Summary and Conclusion

The Golay Hadamard RACH preamble detector can be easily implemented based on the two architectures (1) Nokia despreader based detector (2) Ericsson Efficient Golay correlator based detector.

The Golay Hadamard preamble detector implementation complexity is at most the same as the Nokia Long PN code based preamble detector.

The Golay Hadamard preamble allows several way to optimize the implementation complexity.