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Title:	**BS correlation with antenna pattern**
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1. INTRODUCTION

The aim of this document is to provide a closed form solution to antenna correlation at the base station, based on the SCM model[1], which also takes into account the antenna pattern of the base station antennas. Previous contributions [2] assume an omnidirectional antenna pattern for the BS antennas. Even though only one case is shown in this contribution, the other cases can be easily derived using the same method.

2. DERIVATION OF CORRELATION COEFFICIENT

2.1 Definition of terms

$h_{s,u,l}(t)$: Channel coefficient at time t

s: BS antenna index

u: MS antenna index

l: Path index

t: Time index

$G_{BS}(\theta)$: Antenna response at BS

$$= N_0^{BS} \text{ x } G_{BS,1}(\theta) \text{ x } G_{BS,2}(\theta)$$

$$10\log\left(G_{BS,1}(\theta)\right) = \left(-\min\left[12\left(\frac{\theta}{\theta_{3dB}}\right)^2, A_m\right]\right), \left\{\begin{matrix} \mathrm{A_m} = 20dB \\ \theta_{3dB} = 70\deg. \end{matrix}, \text{for 3 sectors}\right\}$$

$$G_{BS,2}(\theta) = \exp\left[-\sqrt{2}\frac{|\theta-\bar{\theta}|}{\sigma_{BS}}\right]$$

$$\left(N_0^{BS}\right)^{-1} = \int G_{BS,1}(\theta).\exp\left[-\sqrt{2}\frac{|\theta-\bar{\theta}|}{\sigma_{BS}}\right]d\theta = \text{normalization.}$$

$G_{MS}(\theta)$: Antenna response at MS

$$= N_0^{MS} \text{ x } G_{MS,1}(\theta) \text{ x } G_{MS,2}(\theta)$$

$$G_{MS,1}(\beta) = 1 \text{ (uniform)}$$

$$G_{MS,2}(\beta) = \exp\left[-\sqrt{2}\frac{|\beta-\bar{\beta}|}{\sigma_{MS}}\right]$$

$$\left(N_0^{MS}\right)^{-1} = \int \exp\left[-\sqrt{2}\frac{|\beta-\bar{\beta}|}{\sigma_{MS}}\right]d\beta = \text{normalization.} \tag{1}$$

2.2 Correlation Coefficient

Consider the correlation coefficient between two paths with the same delay, impinging on two different base station antennas (s_1, s_2), but emanating from the same mobile antenna (u).

$$\begin{aligned}\rho_{s_1,s_2}^{BS} &= E\left[h_{s_1,u,l}(t)h_{s_2,u,l}^{*}(t)\right] \\ &= \iint G_{BS}(\theta)\exp\left(jk\left(d_{s_1}-d_{s_2}\right)\sin\theta\right)G_{MS}(\beta)\exp\left(jk\left(d_u-d_u\right)\sin\theta\right)d\beta d\theta \\ &= \underbrace{\int G_{MS}(\beta)d\beta}_{=1}\int G_{BS}(\theta)\exp\left(jkd\sin\theta\right)d\theta \qquad (2)\end{aligned}$$

where $d = d_{s_1} - d_{s_2}$,

$$\text{k}=\frac{2\pi}{\lambda}$$

In order to perform the integration above, we first assume that the mean angle of arrival $\bar{\theta}$ falls within the parabolic section of the antenna pattern, and not in the flat section. We also assume a 3 sector case and use the corresponding constants.

The integral in (2) can be divided into the following four portions:

$$\rho_{s_1,s_2}^{BS} = I_1 + I_2 + I_3 + I_4,$$

where

$$\begin{aligned}I_1 &= \int_{-\pi}^{\theta_c} N_0^{BS}.10^{-2}.\exp\left[-\sqrt{2}\frac{\left(\bar{\theta}-\theta\right)}{\sigma_{BS}}\right].\exp\left(jkd\sin\theta\right)d\theta, \\ I_2 &= \int_{-\theta_c}^{\bar{\theta}} N_0^{BS}.10^{\left(-1.2\left(\frac{\theta}{\theta_{3dB}}\right)^2\right)}.\exp\left[-\sqrt{2}\frac{\left(\bar{\theta}-\theta\right)}{\sigma_{BS}}\right].\exp\left(jkd\sin\theta\right)d\theta, \qquad (3) \\ I_3 &= \int_{\bar{\theta}}^{\theta_c} N_0^{BS}.10^{\left(-1.2\left(\frac{\theta}{\theta_{3dB}}\right)^2\right)}.\exp\left[-\sqrt{2}\frac{\left(\theta-\bar{\theta}\right)}{\sigma_{BS}}\right].\exp\left(jkd\sin\theta\right)d\theta, \\ I_4 &= \int_{\theta_c}^{\pi} N_0^{BS}.10^{-2}.\exp\left[-\sqrt{2}\frac{\left(\theta-\bar{\theta}\right)}{\sigma_{BS}}\right].\exp\left(jkd\sin\theta\right)d\theta\end{aligned}$$

Consider the integral over the left extreme flat portion I1:

$$I_1 = \int_{-\pi}^{\theta_c} N_0^{BS}.10^{-2}.\exp\left[-\sqrt{2}\,\frac{\left(\bar{\theta}-\theta\right)}{\sigma_{BS}}\right].\exp\left(jkd\sin\theta\right)d\theta$$

$$= N_0^{BS}.10^{-2}.\exp\left[-\sqrt{2}\,\frac{\bar{\theta}}{\sigma_{BS}}\right]\int_{-\pi}^{\theta_c}\exp\left[\frac{\sqrt{2}\theta}{\sigma_{BS}}\right].\exp\left(jkd\sin\theta\right)d\theta$$

(Expanding $\exp\left(jkd\sin\theta\right)$),

$$= N_0^{BS}.10^{-2}.\exp\left[-\sqrt{2}\,\frac{\bar{\theta}}{\sigma_{BS}}\right]\int_{-\pi}^{\theta_c}\exp\left[\frac{\sqrt{2}\theta}{\sigma_{BS}}\right].\left[\begin{array}{c} J_0\left(kd\right)+ \\ 2\sum_{r=1}^{\infty}J_{2r}\left(kd\right)\cos(2r\theta)+ \\ 2j\sum_{r=0}^{\infty}J_{2r}\left(kd\right)\sin\left(\left(2r+1\right)\theta\right)\end{array}\right]d\theta$$

(Using $\int\exp(ax)\cos(bx)dx = \mathrm{Re}\left[\int\exp\left(\left(a+jb\right)x\right)dx\right]$,

$\int\exp(ax)\sin(bx)dx = \mathrm{Im}\left[\int\exp\left(\left(a+jb\right)x\right)dx\right]$),

$$I_1 = N_0^{BS}.10^{-2}.\exp\left[-\sqrt{2}\,\frac{\bar{\theta}}{\sigma_{BS}}\right].$$

$$\left[\begin{array}{c} J_0\left(kd\right)\left[\dfrac{\exp\left(\dfrac{\sqrt{2}\theta}{\sigma_{BS}}\right)}{\left(\sqrt{2}/\sigma_{BS}\right)}\right]_{-\pi}^{-\theta_c} \\ +2\sum_{r=1}^{\infty}J_{2r}\left(kd\right)\mathrm{Re}\left\{\left[\dfrac{\exp\left(\left(\dfrac{\sqrt{2}}{\sigma_{BS}}+j2r\right)\theta\right)}{\left(\dfrac{\sqrt{2}}{\sigma_{BS}}+j2r\right)}\right]_{-\pi}^{-\theta_c}\right\} \\ +2\sum_{r=0}^{\infty}J_{2r+1}\left(kd\right)\mathrm{Im}\left\{\left[\dfrac{\exp\left(\left(\dfrac{\sqrt{2}}{\sigma_{BS}}+j\left(2r+1\right)\right)\theta\right)}{\left(\dfrac{\sqrt{2}}{\sigma_{BS}}+j\left(2r+1\right)\right)}\right]_{-\pi}^{-\theta_c}\right\}\end{array}\right] \qquad (4)$$

Proceeding in a similar manner, we can compute the integral over the right extreme flat portion as

$$I_4 = N_0^{BS}.10^{-2}.\exp\left[\sqrt{2}\frac{\bar{\theta}}{\sigma_{BS}}\right].$$

$$\left[\begin{array}{c} J_0(kd)\left[\dfrac{\exp\left(-\dfrac{\sqrt{2}\theta}{\sigma_{BS}}\right)}{\left(-\sqrt{2}/\sigma_{BS}\right)}\right]_{\theta_c}^{\pi} \\ +2\sum_{r=1}^{\infty} J_{2r}(kd)\operatorname{Re}\left\{\left[\dfrac{\exp\left(\left(-\dfrac{\sqrt{2}}{\sigma_{BS}}+j2r\right)\theta\right)}{\left(-\dfrac{\sqrt{2}}{\sigma_{BS}}+j2r\right)}\right]_{\theta_c}^{\pi}\right\} \\ +2\sum_{r=0}^{\infty} J_{2r+1}(kd)\operatorname{Im}\left\{\left[\dfrac{\exp\left(\left(-\dfrac{\sqrt{2}}{\sigma_{BS}}+j(2r+1)\right)\theta\right)}{\left(-\dfrac{\sqrt{2}}{\sigma_{BS}}+j(2r+1)\right)}\right]_{\theta_c}^{\pi}\right\} \end{array}\right] \quad (5)$$

Now, we attempt to compute the integrals I_2, I_3 corresponding to the parabolic portions of the antenna pattern:

$$\begin{aligned} I_2 &= \int_{-\theta_c}^{\bar{\theta}} N_0^{BS}.10^{\left(-1.2\left(\frac{\theta}{\theta_{3dB}}\right)^2\right)}.\exp\left[-\sqrt{2}\frac{(\bar{\theta}-\theta)}{\sigma_{BS}}\right].\exp(jkd\sin\theta)d\theta \\ &= \int_{-\theta_c}^{\bar{\theta}} N_0^{BS}.\exp\left[-(1.2\text{x}\log_e 10)\left(\frac{\theta}{\theta_{3dB}}\right)^2\right].\exp\left[-\sqrt{2}\frac{(\bar{\theta}-\theta)}{\sigma_{BS}}\right].\exp(jkd\sin\theta)d\theta \quad (6) \\ &= N_0^{BS}.\exp\left[-\frac{\sqrt{2}\bar{\theta}}{\sigma_{BS}}\right]\int_{-\theta_c}^{\bar{\theta}} \exp\left[\frac{\sqrt{2}\theta}{\sigma_{BS}}-\left(\frac{2.763}{\theta_{3dB}^2}\right)\theta^2 + jkd\sin\theta\right]d\theta \end{aligned}$$

(Using the same Bessel expansion as for I_1, I_4),

$$I_2 = N_0^{BS}.\exp\left[-\frac{\sqrt{2}\bar{\theta}}{\sigma_{BS}}\right].\left[\begin{array}{c} J_0(kd)\displaystyle\int_{-\theta_c}^{\bar{\theta}} \exp\left(\frac{\sqrt{2}\theta}{\sigma_{BS}}-\left(\frac{2.763}{\theta_{3dB}^2}\right)\theta^2\right)d\theta \\ +2\displaystyle\sum_{r=1}^{\infty} J_{2r}(kd)\operatorname{Re}\left\{\int_{-\theta_c}^{\bar{\theta}} \exp\left(\frac{\sqrt{2}\theta}{\sigma_{BS}}-\left(\frac{2.763}{\theta_{3dB}^2}\right)\theta^2 + j2r\theta\right)d\theta\right\} \\ +2\displaystyle\sum_{r=0}^{\infty} J_{2r+1}(kd)\operatorname{Im}\left\{\int_{-\theta_c}^{\bar{\theta}} \exp\left(\frac{\sqrt{2}\theta}{\sigma_{BS}}-\left(\frac{2.763}{\theta_{3dB}^2}\right)\theta^2 + j(2r+1)\theta\right)d\theta\right\} \end{array}\right] \quad (7)$$

Here, we can use the identity

$$\int \exp\left(\left(a+jc\right)x-bx^2\right)dx=\frac{1}{2}\sqrt{\frac{\pi}{b}}\exp\left(\frac{\left(a+jc\right)^2}{4b}\right)erf\left(\sqrt{b}x-\frac{\left(a+jc\right)}{2\sqrt{b}}\right) \tag{8}$$

to compute the integral I_2, where the function $erf\left(z\right)$, where z is a complex quantity,can be computed using the series expansions in [3].

Proceeding similarly, we find that

$$I_3=N_0^{BS}.\exp\left[\frac{\sqrt{2}\bar{\theta}}{\sigma_{BS}}\right].\left[\begin{array}{c} J_0\left(kd\right)\int_{\bar{\theta}}^{\theta_c}\exp\left(-\frac{\sqrt{2}\theta}{\sigma_{BS}}-\left(\frac{2.763}{\theta_{3dB}^2}\right)\theta^2\right)d\theta \\ +2\sum_{r=1}^{\infty}J_{2r}\left(kd\right)\mathrm{Re}\left\{\int_{\bar{\theta}}^{\theta_c}\exp\left(-\frac{\sqrt{2}\theta}{\sigma_{BS}}-\left(\frac{2.763}{\theta_{3dB}^2}\right)\theta^2+j2r\theta\right)d\theta\right\} \\ +2\sum_{r=0}^{\infty}J_{2r+1}\left(kd\right)\mathrm{Im}\left\{\int_{\bar{\theta}}^{\theta_c}\exp\left(-\frac{\sqrt{2}\theta}{\sigma_{BS}}-\left(\frac{2.763}{\theta_{3dB}^2}\right)\theta^2+j\left(2r+1\right)\theta\right)d\theta\right\}\end{array}\right] \tag{9}$$

The normalization function N_0^{BS} can be found as the sum of four integrals :

$$N_0^{BS}=\left(N_{01}^{BS}+N_{02}^{BS}+N_{03}^{BS}+N_{04}^{BS}\right)^{-1}$$

$$N_{01}^{BS}=\int_{-\pi}^{\theta_c}10^{-2}.\exp\left[-\sqrt{2}\frac{\left(\bar{\theta}-\theta\right)}{\sigma_{BS}}\right]d\theta=10^{-2}\exp\left[-\frac{\sqrt{2}\bar{\theta}}{\sigma_{BS}}\right]\left[\frac{\exp\left(\frac{\sqrt{2}\theta}{\sigma_{BS}}\right)}{\left(\sqrt{2}/\sigma_{BS}\right)}\right]_{-\pi}^{-\theta_c}$$

$$N_{02}^{BS}=\int_{-\theta_c}^{\bar{\theta}}10^{\left(-1.2\left(\frac{\theta}{\theta_{3dB}}\right)^2\right)}.\exp\left[-\sqrt{2}\frac{\left(\bar{\theta}-\theta\right)}{\sigma_{BS}}\right]d\theta=\exp\left[-\sqrt{2}\frac{\bar{\theta}}{\sigma_{BS}}\right]\int_{-\theta_c}^{\bar{\theta}}\exp\left(\frac{\sqrt{2}\theta}{\sigma_{BS}}-\left(\frac{2.763}{\theta_{3dB}^2}\right)\theta^2\right)d\theta$$

$$N_{03}^{BS}=\int_{\bar{\theta}}^{\theta_c}10^{\left(-1.2\left(\frac{\theta}{\theta_{3dB}}\right)^2\right)}.\exp\left[-\sqrt{2}\frac{\left(\theta-\bar{\theta}\right)}{\sigma_{BS}}\right]d\theta=\exp\left[\sqrt{2}\frac{\bar{\theta}}{\sigma_{BS}}\right]\int_{\bar{\theta}}^{\theta_c}\exp\left(-\frac{\sqrt{2}\theta}{\sigma_{BS}}-\left(\frac{2.763}{\theta_{3dB}^2}\right)\theta^2\right)d\theta$$

$$N_{04}^{BS}=\int_{\theta_c}^{\pi}10^{-2}.\exp\left[-\sqrt{2}\frac{\left(\theta-\bar{\theta}\right)}{\sigma_{BS}}\right]d\theta=10^{-2}\exp\left[\frac{\sqrt{2}\bar{\theta}}{\sigma_{BS}}\right]\left[\frac{\exp\left(-\frac{\sqrt{2}\theta}{\sigma_{BS}}\right)}{\left(-\sqrt{2}/\sigma_{BS}\right)}\right]_{\theta_c}^{\pi} \tag{10}$$

The terms N_{02}^{BS}, N_{03}^{BS} are again found using the solution from (8) with real arguments.

2.3 Comparison with numerical integration

The correlation coefficient ρ_{s_1,s_2}^{BS} thus computed using the closed form expressions in equations (3)-(10) is now compared with the values generated using numerical integration. Two sets of curves are given below. Figure 1 shows the results for large per-path angular spread $(\sigma_{BS} = 35\deg.)$, and Figure 2 shows results for a low angular spread $(\sigma_{BS} = 5\deg.)$. The mean angle of departure for both cases is $\overline{\theta} = 20\deg$.

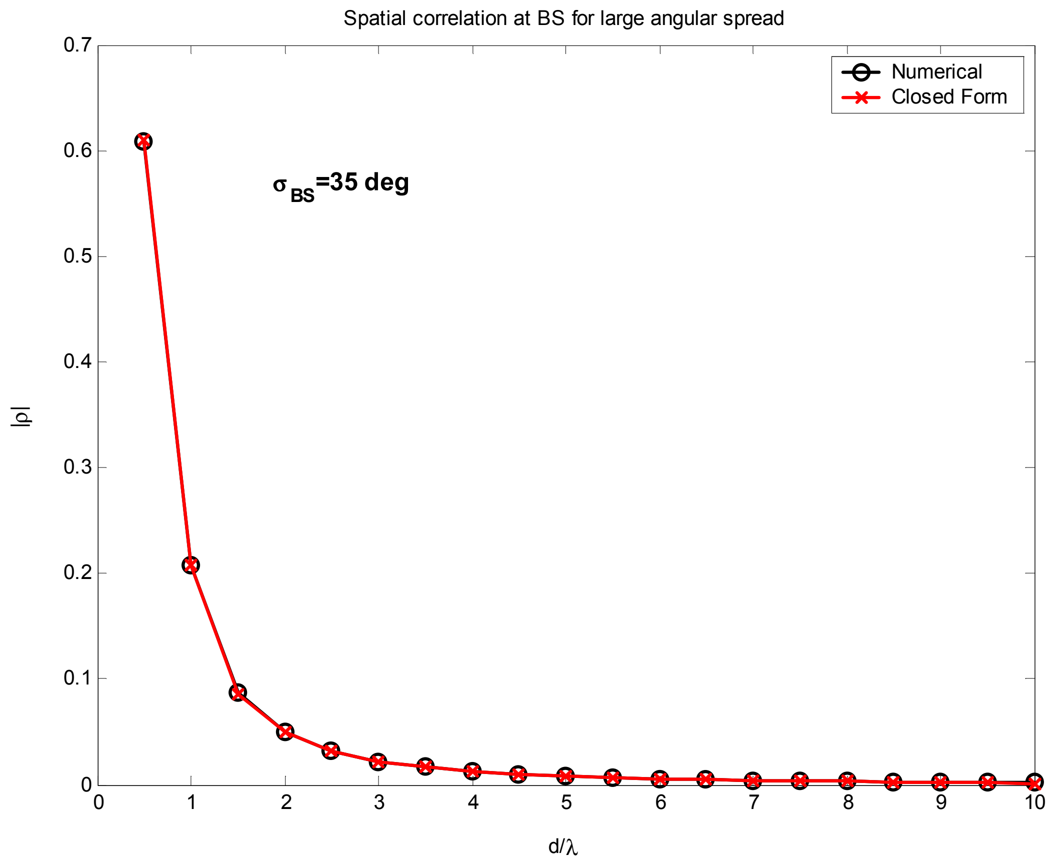

Figure 1. Correlation coefficient at large angular spreads

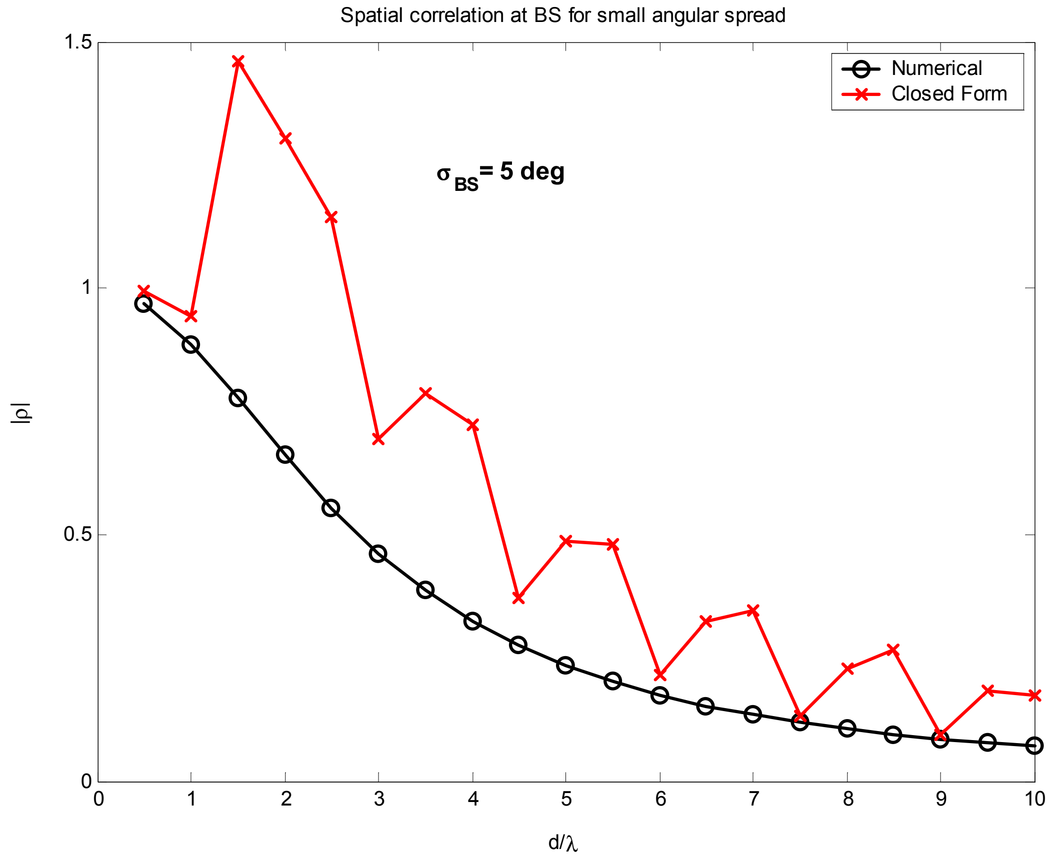

Figure 2. Correlation coefficient at small angular spreads

It can be seen that the closed form expression varies significantly from the numerical solution for small angular spreads. However, at the BS, mostly smaller angular spreads have been defined. Hence the expressions cannot be used as such. In the following subsection, we provide an approximation which rectifies this discrepancy.

2.4 Approximation for small σ_{BS}

On further investigation, it was found that the source of the problem for smaller angular spreads lies in the expression in equation (8). Specifically, in $\frac{1}{2}\sqrt{\frac{\pi}{b}}\exp\left(\frac{(a+jc)^2}{4b}\right)erf\left(\sqrt{b}x-\frac{(a+jc)}{2\sqrt{b}}\right)$, we have $a=\pm\frac{\sqrt{2}}{\sigma_{BS}}$, $b=\left(\frac{2.763}{\theta_{3dB}^2}\right)$, $c=m$. For small σ_{BS}, $(a+jc)\gg b$, the expression computes to $\gamma\left(\xi^+-\xi^-\right)$, where $\gamma\to\infty$, and $\left(\xi^+-\xi^-\right)\sim 0$.

In order to solve this problem, an approximation was used, wherein, for small angular spreads, we assume a flat antenna pattern instead of a parabolic function, with the gain being equal to the value of the gain at the mean angle of departure. That is to say,

$$G_{BS,1}(\theta)\approx 10^{\left(-1.2\left(\frac{\bar{\theta}}{\theta_{3dB}}\right)^2\right)},\ \text{for } \theta\in[-\theta_c,\theta_c] \tag{11}$$

Using the above approximation simplifies the integrals I_2, I_3 in equations (7), (9) to a form similar to those of I_1, I_4 respectively. It also simplifies the normalization terms $N_{0,2}^{BS}, N_{0,3}^{BS}$ in equations (10) to a form similar to $N_{0,1}^{BS}, N_{0,4}^{BS}$ respectively. Using these modifications, the closed form expression for correlation is plotted again in Figure 3. It is found that the modified closed form is in close agreement with those obtained by numerical integration.

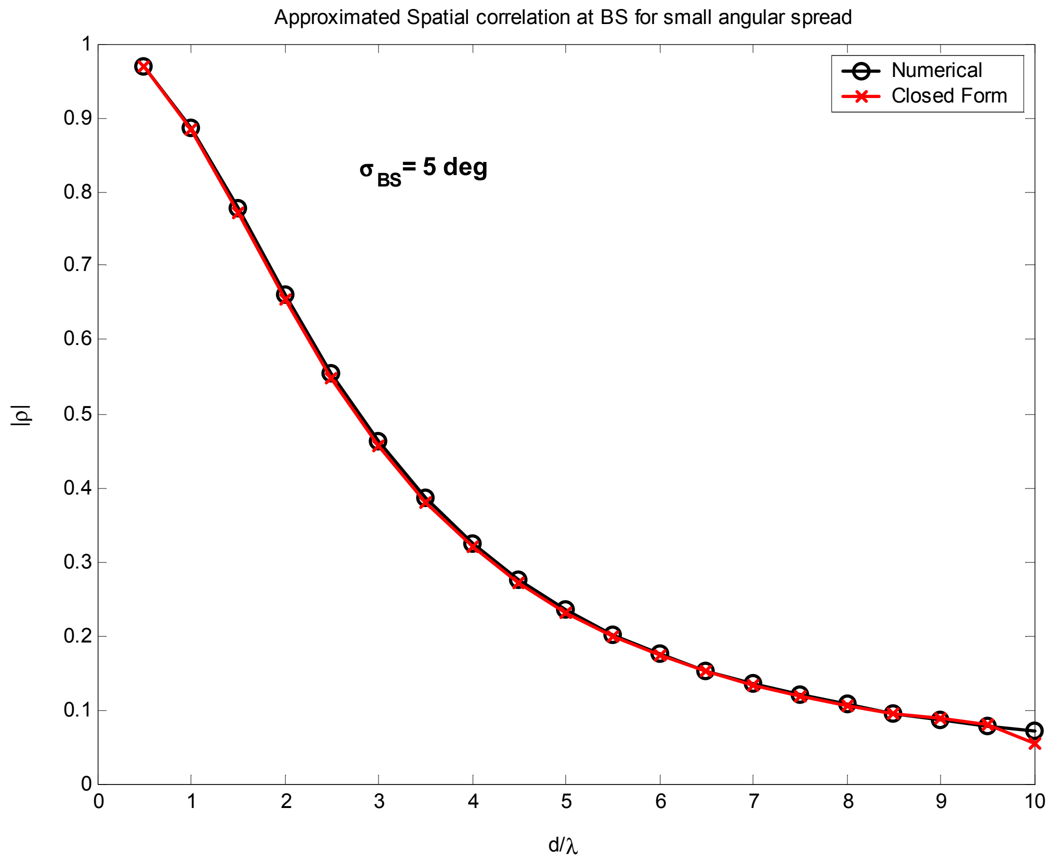

Figure 3. Correlation coefficient at small angular spreads with approximated closed form

For further validation, in table the values listed for reference in the "BS" portion of table 2-2 of the SCM Text [1], are compared with those obtained from the closed form solution

	Antenna Spacing	AS deg.	AOA (deg.)	Numerical soln.: Correlation (magnitude)	Closed form soln.: Correlation (magnitude)	Numerical soln.: Complex Correlation	Closed form: Complex Corr.
BS	0.5 λ	5	20	0.9688	0.9683	0.4743+ 0.8448i	0.4640 + 0.8499i

	0.5 λ	2	50	0.9975	0.9975	-0.7367+ 0.6725i	-0.7390 + 0.6700i
	4 λ	5	20	0.3224	0.3198	-0.2144+ 0.2408i	-0.2203 + 0.2318i
	4 λ	2	50	0.8624	0.8631	0.8025+ 0.3158i	0.7954 + 0.3350i
	10 λ	5	20	0.0704	0.0544	-0.0617+ 0.034i	-0.0352 + 0.0415i
	10 λ	2	50	0.5018	0.4502	-0.2762- 0.4190i	-0.2392 - 0.3814i

3. CONCLUSIONS

Closed form expressions are derived for correlatin coefficients in the proposed spatial channe model. These expressions can be used to estimate the correlations that can be expected in the generated channel.

4. REFERENCES

[1] SCM-083, Spatial Channel Model Text Description v2.1, Dec. 2002.
[2] SCM-096R1, Correlation computation of SCM.
[3] Abramovitz and Stegun, Integral Tables.