3GPP TSG-GERAN Meeting #18 **Tdoc GP-040356**
Reykjavik (Iceland), 2-6 February 2004 **Agenda item: 7.1.5.7**
Source: Siemens

Performance of p-t-M Retransmission Strategies for MBMS

1 Introduction

At GERAN#17 in [2] the Common Feedback Channel (CFCH) was proposed to convey NACK messages from the MS to the BTS in order to indicate lost RLC blocks [5]. Furthermore, a simple retransmission scheme (p-t-M) based on NACKs signaled on the CFCH was proposed and first promising performance results were already presented.

In [1] a modified retransmission scheme based on incremental redundancy (p-t-M-IR) and outer RS-coding, which exploits the CFCH in the same manner as the p-t-M retransmission scheme, was proposed. In addition, theoretical performance analysis for the p-t-p, p-t-M and the p-t-M-IR retransmissions schemes have been presented. The throughput degeneration with increasing number of receiving MSs was compared. It was shown that

- significant gains in terms of throughput can be expected by introducing and exploiting the CFCH,
- that p-t-p retransmissions cannot provide sufficient efficiency to support an acknowledged mode bearer within MBMS for an increased number of users to be served in a cell as typical in common scenarios, e.g. football stadium,
- that the p-t-M-IR scheme outperforms the p-t-p retransmission scenario, as well as the plain p-t-M retransmission scenario significantly,
- that the p-t-M-IR throughput performance only slightly depends on the number of served user terminals in the contrast to the other schemes, which degenerate significantly with increasing number of users.

However, in [1] for all presented schemes it was assumed that the BTS retransmits lost RLC blocks in persistent mode which is not required in the MBMS framework.

Therefore, in this contribution the “persistent mode” assumption is replaced by a limited number of retransmissions. Results for throughput and error rate performance for a limited number of retransmissions are presented. The new results confirm the results in [1], and, in addition, it is shown

- that the p-t-M-IR approach outperforms the p-t-p retransmission scenario, as well as the plain p-t-M retransmission scenario, and,
- that the throughput is only slightly depending on the number of served user terminals in the contrast to the other schemes, which degenerate significantly with increasing number of users,
- that the residual error rate is significantly lower for the p-t-M-IR approach when compared to the other schemes.

2 Retransmission Strategies

In this section we review very briefly the previously investigated Automatic Repeat reQuest (ARQ) schemes [1],[2]. Already presented in [1] for unlimited amount of retransmissions, we extend the

framework in this document by limiting the number of retransmissions, and, therefore, also limiting the maximum number of incremental redundancy packets.

In the next subsections the following notation is used:

M	total number of served MSs
p	RLC block error probability (depends on C/I)[1]
L(CS)	payload size of applied RLC coding scheme in byte depending on coding scheme CS
R	maximum number of allowed retransmissions
r	average number of redundancy blocks per information block
$\eta(M,p)$	average throughput in kbit/s for single slot transmission

p-t-p Retransmissions

The p-t-p retransmission scheme includes the following steps:

1. p-t-M connection to all MS is established.
2. Data (RLC blocks) is broadcasted in best-effort manner.
3. MSs which have detected lost RLC blocks establish p-t-p connections in acknowledged mode and request all lost packets individually.
4. All lost packets are retransmitted in separate p-t-p connections to the terminals.
5. Retransmissions for a specific RLC block are stopped after R retransmission attempts

In Annex A the derivation of the throughput and residual error rate for this scenario is given. The average throughput of the p-t-p retransmission scenario assuming equal RLC block error probability for all user terminals is given by

$$\eta(M,p) = \frac{0.4 \cdot L(\mathrm{CS})}{1 + M\left(\sum_{t=0}^{R-1} t p^t (1-p) + R\left(1 - \sum_{i=0}^{R-1} p^i (1-p)\right)\right)} \text{ in kbit/s.}$$

The residual RLC error rate results in

$$p_{RLC} = p^{R+1},$$

and, therefore, decreases exponentially with the number of allowed retransmissions.

p-t-M Retransmission

The p-t-M retransmission scheme includes the following steps:

1. p-t-M connection to all MS is established.
2. Data (RLC blocks) is broadcasted in best-effort manner.
3. MSs which have detected lost RLC blocks send NACK on the CFCH [5] (ACKs are not sent)

[1] For the time being we assume that all users experience the same loss probability which somehow reflects a worst case scenario.

4. BTS re-broadcasts lost RLC blocks (p-t-M mode).
5. Retransmissions for a specific RLC block are stopped at the latest after R transmission attempts.

According to Annex A and considering the setup and notation already introduced, the average throughput of the p-t-M retransmission scenario results in

$$\eta(M,p) = \frac{0.4 \cdot L(\mathrm{CS})}{\sum_{t=1}^{R} t \cdot \left(\left[1-p^{t}\right]^{M} - \left[1-p^{t-1}\right]^{M} \right) - (R+1)\left(1-\left(1-p^{R}\right)^{M}\right)} \text{ in kbit/s.}$$

The residual RLC block was found to be

$$p_{RLC} = p^{R+1},$$

and decreases exponentially with the number of allowed retransmissions.

p-t-M Retransmissions exploiting incremental redundancy (p-t-M-IR)

The p-t-M-IR retransmission scheme includes the following steps:

1. p-t-M connection to all MS is established
2. n-k redundancy blocks are generated from k RLC blocks as proposed in [3],[4] using an (n,k) Reed-Solomon code, where n=(r+1)k, with r denoting the number of redundancy blocks per information block (on average).
3. RLC blocks are broadcasted in p-t-M mode
4. MS which could not receive all k RLC blocks correctly indicate this on the CFCH [5] (ACKs are not send)
5. After broadcasting k information blocks, the BTS broadcasts additional blocks, if required, from the set of n-k redundancy blocks.
6. Transmission of redundancy blocks is stopped at the latest when the redundancy limit is exceeded.

The average throughput of the p-t-M-IR retransmission scenario for the case where one redundancy block is broadcasted after each negative acknowledgement can be determined as

$$\eta(M,p) = \frac{0.4 \cdot L(\mathrm{CS}) \cdot k}{E\{T\}} \text{ in kbit/s,}$$

with

$$E\{T\} = \sum_{t=k}^{k(r+1)-1} t \cdot \left\{ \left(1 - \sum_{i=0}^{k-1} \binom{t}{i} (1-p)^{i} p^{t-i}\right)^{M} - \left(1 - \sum_{i=0}^{k-1} \binom{t-1}{i} (1-p)^{i} p^{t-1-i}\right)^{M} \right\} +$$

$$+k(r+1)\left\{1 - \sum_{j=k}^{k(r+1)-1} \left(\left(1 - \sum_{i=0}^{k-1} \binom{j}{i} (1-p)^{i} p^{j-i}\right)^{M} - \left(1 - \sum_{i=0}^{k-1} \binom{j-1}{i} (1-p)^{i} p^{j-1-i}\right)^{M} \right)\right\}.$$

The residual RLC block error rate results in

$$p_{RLC} = \frac{1}{n} \sum_{i=k\cdot r+1}^{k(r+1)} i \binom{k(r+1)}{i} p^i (1-p)^{k(r+1)-i} .$$

3 Performance Evaluation

In this section the achievable throughput and the residual error rate for single radio time-slot of all presented retransmission scenarios are compared. For this, we have used RLC block loss rates over a TU03 channel with frequency hopping at different C/I. This is summarized in Table 1 along with the used payload size *L* for each GPRS coding scheme (CS). Throughput, RLC block error rate and SDU error rate were determined analytically. Simulations will be carried out to confirm these results.

Table 1 Payload size and RLC/MAC block loss rates at different C/I, different coding schemes with frequency hopping.

	Payload size *L*	7.5 dB FH	10 dB FH	12.5 dB FH
CS 1	20 bytes	0.119	0.030	0.008
CS 2	30 bytes	0.357	0.150	0.051
CS 3	36 bytes	0.502	0.271	0.108
CS 4	50 bytes	0.912	0.777	0.598

In Fig. 1 the achievable throughput for p-t-p retransmissions, plain p-t-M retransmissions and p-t-M-IR retransmissions over the number of users *M* is shown for C/I=7.5dB, coding scheme CS1 and ideal frequency hopping.

It can be observed that

- all schemes degenerate in terms of average throughput with increasing number of users,
- degeneration of p-t-M scheme is significantly lower (2-3 times higher throughput compared to p-t-p case for *M*=50 users)
- p-t-M-IR scheme outperforms other schemes significantly in terms of
  - average throughput (5.5 times higher than p-t-p, doubled compared to p-t-M @ *M*=50),
  - throughput degeneration due to increasing users,
  - throughput degeneration due to increasing number of allowed retransmissions,
- Extreme low throughput for the p-t-p retransmission scheme of 1 kbit/s does not seem to be sufficient to define an MBMS acknowledged bearer with the p-t-p scheme.

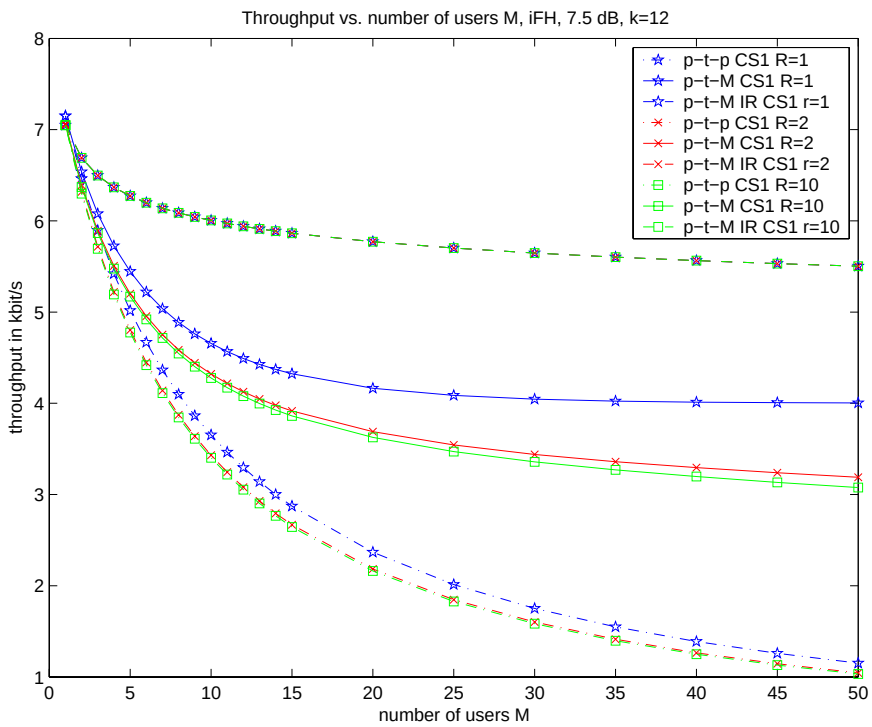

Fig. 1.: Throughput of p-t-p, p-t-M and p-t-M-IR retransmissions over number of users M for C/I=7.5 dB and CS1 for different number of allowed retransmissions.

In Fig. 2 the same curves as in Fig. 1, but for an increased number of users (M =1…2000) are shown. It can be observed that

- the p-t-p retransmissions scheme cannot support a large number of users,
- the p-t-M and the p-t-M-IR scheme are able to support a high number of users,
- p-t-M-IR scheme outperforms the p-t-p and the p-t-M retransmission schemes.

As the maximum amount of redundancy per information package is limited residual RLC block errors are expected. In Fig. 3 the residual RLC block error rate is shown over the number of allowed retransmissions R or the average number of redundancy blocks r per block, respectively. It can be observed that

- the residual RLC block error rate is equal for the p-t-p and the p-t-M scheme,
- the p-t-M-IR retransmission scheme provides a significantly lower residual error rate,
- the residual RLC block error rate does in general not depend on the number of users.

In Fig. 4 the residual SDU error rate (IP-packet error rate) for a SDU frame size of S=500 byte is depicted. It can be observed that

- the residual SDU frame error rate of the p-t-p scheme and the p-t-M scheme are equal,
- the residual SDU frame error rate of the p-t-M-IR scheme is significantly lower,
- to guarantee $p_{SDU} < 10^{-3}$ for p-t-p and p-t-M R>4 is required, whereas for p-t-M-IR for r=1 $p_{SDU} < 10^{-5}$.

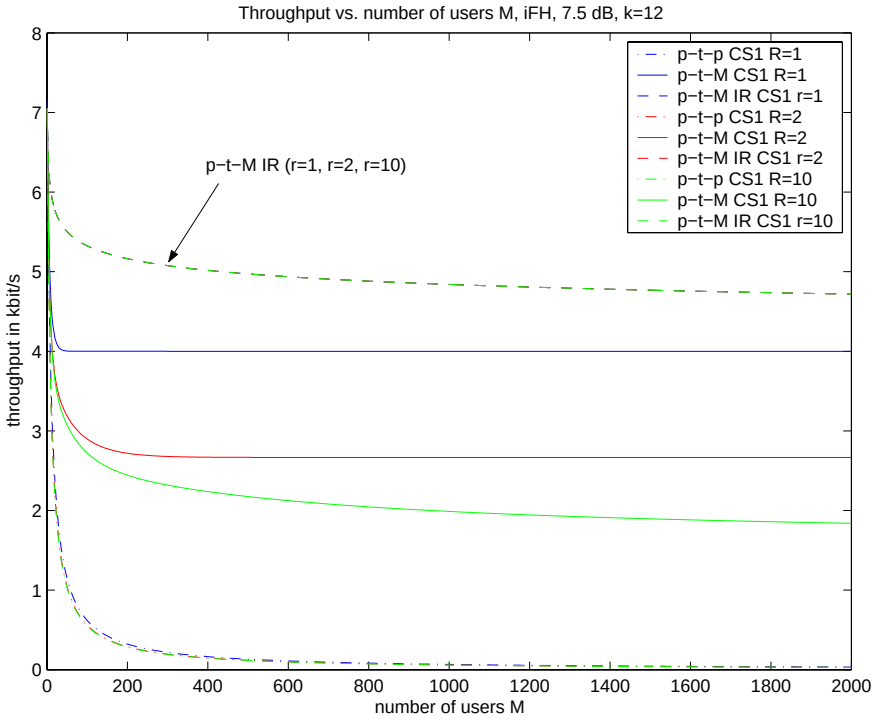

Fig. 2 Throughput of p-t-p, p-t-M, p-t-M-IR for k=12,C/I=7.5 dB and CS1.

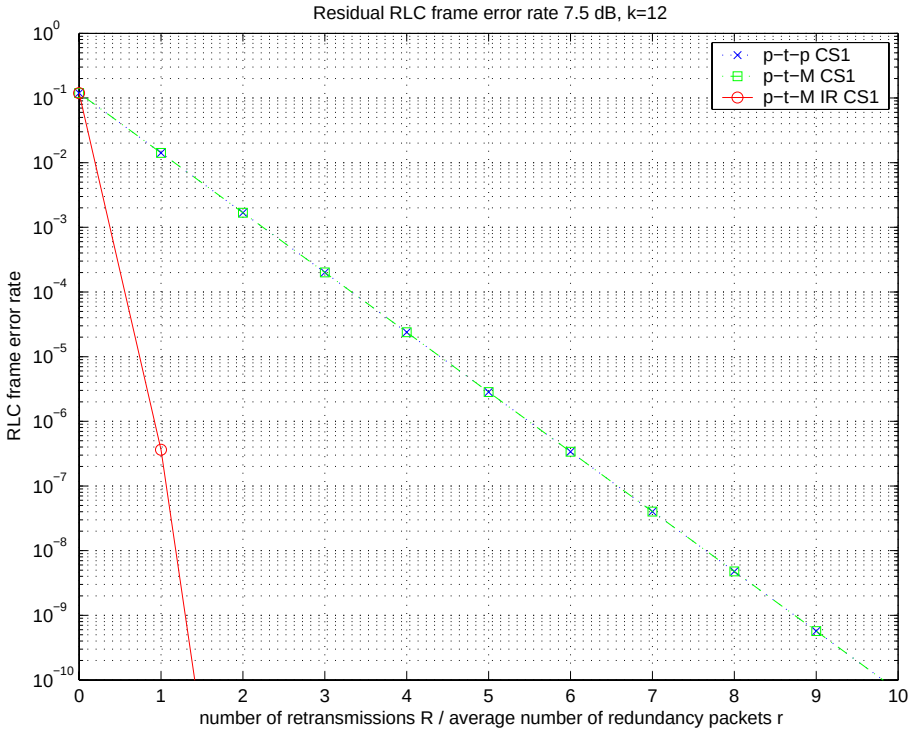

Fig. 3 Residual RLC block error rate for C/I=7.5 dB and CS1.

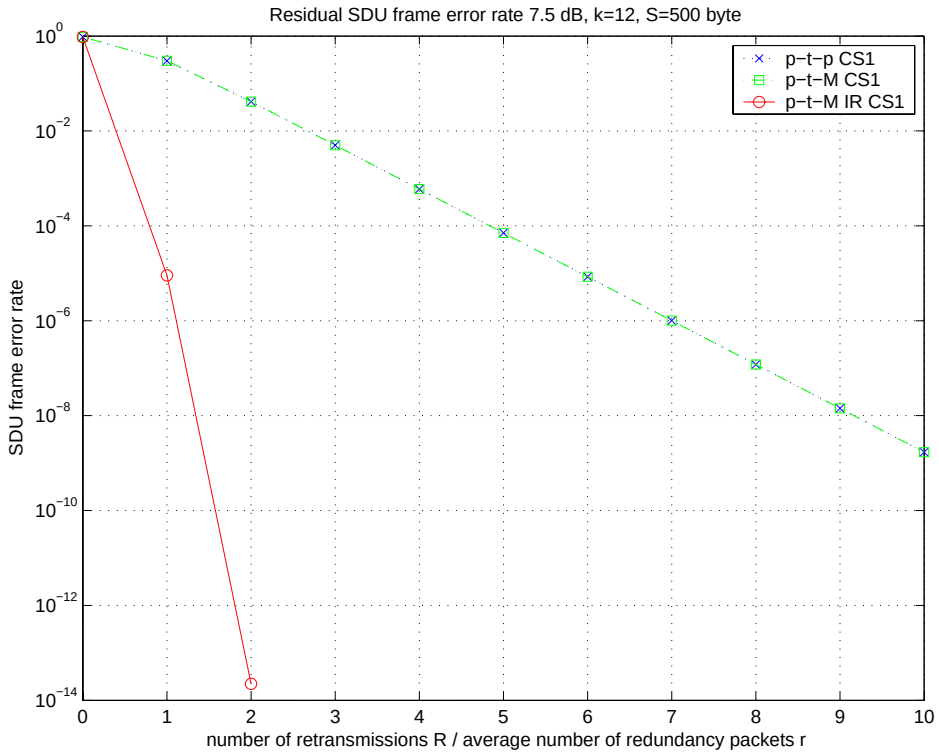

Fig. 4 Residual SDU frame error rate forC/I=7.5 dB and CS1.

4 Conclusions

In this document we compared several retransmission strategies to define an MBMS acknowledged mode bearer. Therefore, we extended the investigations in [2] already presented at GERAN#17. We compared p-t-p retransmissions with p-t-M [1] and showed that significant gains are obtained in terms of throughput by introducing and exploiting the CFCH [5]. It was shown that the residual frame error rate is equal for both schemes and that p-t-p retransmissions cannot provide sufficient efficiency to define an acknowledged mode bearer within MBMS for an increased number of users to be served in a cell.

It was also shown that p-t-M-IR approach outperforms the p-t-p retransmission scenario, as well as the plain p-t-M retransmission scenario significantly in terms of throughput as well as the residual error rate, and, that the throughput performance is only slightly depending on the number of served user terminals in the contrast to the other schemes, which degenerate significantly with increasing number of users. The residual frame error rate was shown to be independent of the number of users, and significantly lower for the p-t-M-IR scheme.

5 References

[1] GP-032647, "Retransmission Strategies for MBMS", Siemens, GERAN #17, Budapest, Hungary, 17-21 November 2003.

[2] GP-032604, "p-t-M with feedback for MBMS delivery: overview", Siemens, GERAN #17, Budapest, Hungary, 17-21 November 2003.

[3] GP-032614, "Outer coding in the RLC for MBMS", Siemens, GERAN #17, Budapest, Hungary, 17-21 November 2003.

[4] GP-032613, "Complexity estimate for Outer Coding in the RLC layer: Encoding and Decoding with Reed-Solomon codes", Siemens, GERAN#17, Budapest, Hungary, 17-21 November 2003.

[5] GP-0400301, "Common Feedback Channel for MBMS delivery", Siemens, GERAN #18, Reykjavik, Iceland, 2-6 February 2004.

6 Annex A

In this section the relative throughput and the residual error rate of the presented ARQ schemes is derived. In contrast to Tdoc GP-032647 [1] in these derivations a limited number of retransmissions is considered. For reasons of simplicity, in the following an RLC block is denoted by "packet". For all derivations we assume that a received packet at receiver m is erroneous with probability p_m and received correctly with probability $1-p_m$. Furthermore, it is assumed that packet losses occur statistically independent (ideal frequency hopping) and that packet losses occur independently among receivers.

P-t-P Retransmissions

In order to calculate the throughput of the p-t-p retransmission scheme the transmission of a single packet from the BTS to M user terminals is considered. Let T_m be a random variable denoting the overall number of performed retransmission attempts (channel uses) in order to transmit one packet to **a single** user m, with $T_m \in \{0,1,...,R\}$ and $m \in \{1,2,...,M\}$, where R denotes the maximum number of allowed retransmissions for one packet. We consider that the initial transmission attempt for this packet is within the multicast session and all retransmissions take place in the separate p-t-p session for this specific user terminal. Hence, the probability that exactly t retransmissions are performed is

$$\Pr\{T_m = t\} = \begin{cases} p_m^t(1-p_m) & t < R \\ 1-\sum_{i=0}^{R-1} p_m^i(1-p_m) & t = R \end{cases},$$

with $t \in \{0,1,...,R\}$.

Consequently, the expected value $E\{T_m\}$ for the number of retransmissions at receiver m results in

$$E\{T_m\} = \sum_{t=0}^{R} t \Pr\{T_m = t\} = \sum_{t=0}^{R-1} t p_m^t(1-p_m) + R\left(1-\sum_{i=0}^{R-1} p_m^i(1-p_m)\right),$$

which allows to give the relative throughput $\eta(\mathbf{p})$ of the p-t-p retransmission scenario as

$$\eta(\mathbf{p}) = \frac{1}{1+\sum_m E\{T_m\}} = \frac{1}{1+\sum_m \left\{\sum_{t=0}^{R-1} t p_m^t (1-p_m) + R\left(1-\sum_{i=0}^{R-1} p_m^i (1-p_m)\right)\right\}},$$

with $\mathbf{p} = (p_1 \quad p_2 \quad \ldots \quad p_M)$.

If the packet could neither be delivered within the broadcast session, nor within the p-t-p repair session to user m, the packet transmission failed exactly R+1 times. Hence, the residual packet error rate (RLC block error rate) at user m is given by

$$p_{res,m} = p_m^{R+1}.$$

P-t-M Retransmissions

We consider M terminals in the cell. Each time a broadcasted packet could still not be received by a user terminal the latter sends a negative acknowledgment, denoted by *nak*, to the BTS. In the case of a correctly received packet no message is sent by the terminals. The BTS receives the superposition of all *naks,* sent by all terminals which could still not receive the packet correctly. We denote the superposition of all *naks,* which can also be interpreted as an OR connection of all send *naks*, as *NAK*. In other words, if the BTS receives a *NAK*, at least one user terminal sent a *nak*.

Let t denote the number of packet replicas broadcasted to the receivers, which implies t-1 retransmissions of a certain packet, and let R denote the maximum allowed number of retransmissions. Let $p_{\mathrm{n},m}(t)$ denote the probability that a single receiver m sends a *nak* after it received t replicas of the same packet, i.e. the probability of unsuccessful reception of the current packet after t transmission attempts. Hence, $p_{\mathrm{n},m}(t)$ is given by

$$p_{\mathrm{n},m}(t) = p_m^t.$$

Let $p_{\mathrm{N}}(t)$ denote the probability of a *NAK* reception at the BTS after t broadcast attempts which equals to the probability that at least one user terminal sends a *nak* after t transmissions. Thus, $p_{\mathrm{N}}(t)$ results in

$$p_{\mathrm{N}}(t) = 1-\prod_m \left[1-p_{\mathrm{n},m}(t)\right] = 1-\prod_m \left[1-p_m^t\right].$$

Let T be a random variable denoting the overall number of performed broadcast attempts (channel uses), including the first transmission and all retransmissions, to broadcast one packet correctly to **all** users with $T = 1, 2, \ldots, R+1$. Let $\Pr\{T > t\}$ denote the probability that more than t transmissions are required to deliver the packet to all receivers correctly, which is given by the probability that after t transmissions a *NAK* is received and, thus there are still user terminals requesting retransmissions. Hence, $\Pr\{T > t\}$ yields

$$\Pr\{T > t\} = p_{\mathrm{N}}(t) = 1-\prod_m \left[1-p_m^t\right].$$

Consequently, the probability $\Pr\{T = t\}$ that exactly t replica transmissions are performed results in

$$\begin{aligned}\Pr\{T=t\} &= \Pr\{T>t-1\}-\Pr\{T>t\}= \\ &= p_{\mathrm{N}}(t-1)-p_{\mathrm{N}}(t)= \\ &= 1-\prod_m\left[1-p_m^{t-1}\right]-\left[1-\prod_m\left[1-p_m^{t}\right]\right]= \\ &= \prod_m\left[1-p_m^{t}\right]-\prod_m\left[1-p_m^{t-1}\right]\end{aligned}$$

for $t<R+1$. Ultimatively, we are interested in the probability that R retransmission attempts have been performed, i.e.

$$\Pr\{T=R+1\}=\Pr\{T>R\}=p_{\mathrm{N}}(R)=1-\prod_m\left[1-p_m^{R}\right].$$

Consequently, the expected number of broadcast transmissions attempts for a certain packet results in

$$E\{T\}=\sum_{t=1}^{R+1}t\cdot\Pr\{T=t\}=\sum_{t=1}^{R}t\cdot\left(\prod_m\left[1-p_m^{t}\right]-\prod_m\left[1-p_m^{t-1}\right]\right)+(R+1)\left(1-\prod_m\left[1-p_m^{R}\right]\right),$$

which allows estimating the relative throughput $\eta(\mathbf{p})$ of the p-t-m retransmission scenario as

$$\eta(\mathbf{p})=\frac{1}{E\{T\}}=\frac{1}{\sum_{t=1}^{R}t\cdot\left(\prod_m\left[1-p_m^{t}\right]-\prod_m\left[1-p_m^{t-1}\right]\right)+(R+1)\left(1-\prod_m\left[1-p_m^{R}\right]\right)}.$$

If the packet could neither be delivered to user m within the first broadcast attempt, nor within the p-t-M retransmission attempts, the packet transmission failed exactly R+1 times. Hence, the residual packet error rate (RLC block error rate) at user m is given by

$$p_{res,m}=p_m^{R+1}.$$

P-t-M Incremental Redundancy with Reed-Solomon Codes

Suppose a systematic (n,k) Reed-Solomon code is used to create n-k redundancy packets and suppose that each packet is in correspondence to one Reed-Solomon code symbol. First, we consider k packets to be broadcasted to the user terminals. Second, after k packets were broadcasted, the BTS sends redundancy packets if required, but at most n-k redundancy packets. Let t denote the total number of packets broadcasted for one code word. User terminals which could not recover all k information packets correctly after t transmission attempts send *naks* to the BTS, as already introduced in the previous subsection. The BTS receives a corresponding *NAK* and will transmit the next symbol (packet) from the code word. Note, for $t<k$, none of the user terminals will be able to receive k packets, as only $t<k$ packets were broadcasted. To compare this scheme in a fair way with the other presented schemes, let us define r as the average number of redundancy packets available per information packet. Due to the restrictions by the RS code the maximum number of retransmissions should be limited to

$$r\le r_{\max}:=\frac{n-k}{k}.$$

In order to calculate the throughput we are interested in the expected value of the overall number of packet transmissions needed to recover all k information packets correctly at all users. Let $p_{\mathrm{n},m}(t)$ denote the probability that a single receiver sends a *nak* after it received t packets, i.e. the probability of unsuccessful reception of k packets after t transmission attempts. Hence, $p_{\mathrm{n},m}(t)$ is given by

$$p_{\mathrm{n},m}(t)=\begin{cases} 1 & t<k \\ \sum_{i=0}^{k-1}\binom{t}{i}(1-p_m)^i\, p_m^{t-i} & t\geq k. \end{cases}$$

Let $p_{\mathrm{N}}(t)$ denote the probability of a *NAK* reception at the BTS after t broadcast attempts which equals the probability that at least one user terminal sends a *nak* after t transmissions. Thus, $p_{\mathrm{N}}(t)$ can be written as

$$p_{\mathrm{N}}(t)=1-\prod_m\left[1-p_{\mathrm{n},m}(t)\right]=\begin{cases} 1 & t<k \\ 1-\prod_m\left[1-\sum_{i=0}^{k-1}\binom{t}{i}(1-p_m)^i\, p_m^{t-i}\right] & t\geq k. \end{cases}$$

Let T be a random variable denoting the overall number of performed broadcast attempts (channel uses), including the k systematic transmissions and all redundancy packet transmissions, performed in order to broadcast k packets to **all** users. Let $\Pr\{T>t\}$ denote the probability that more than t transmissions are required to recover all k packet at all receivers. This can equivalently be expressed as the probability that after t transmissions a *NAK* is received yielding

$$\Pr\{T>t\}=p_{\mathrm{N}}(t)=\begin{cases} 1 & t<k \\ 1-\prod_m\left[1-\sum_{i=0}^{k-1}\binom{t}{i}(1-p_m)^i\, p_m^{t-i}\right] & t\geq k. \end{cases}.$$

Hence, the probability $\Pr\{T=t\}$ that exactly t transmissions are necessary with $k\leq t\leq n$ is obtained by

$$\begin{aligned}\Pr\{T=t\}&=\Pr\{T>t-1\}-\Pr\{T>t\}=\\ &=p_{\mathrm{N}}(t-1)-p_{\mathrm{N}}(t)=\\ &=1-\prod_m\left[1-\sum_{i=0}^{k-1}\binom{t-1}{i}(1-p_m)^i\, p_m^{t-1-i}\right]-\left[1-\prod_m\left[1-\sum_{i=0}^{k-1}\binom{t}{i}(1-p_m)^i\, p_m^{t-i}\right]\right]=\\ &=\prod_m\left[1-\sum_{i=0}^{k-1}\binom{t}{i}(1-p_m)^i\, p_m^{t-i}\right]-\prod_m\left[1-\sum_{i=0}^{k-1}\binom{t-1}{i}(1-p_m)^i\, p_m^{t-1-i}\right].\end{aligned}$$

For $t<k$ it is obvious that $\Pr\{T=t\}=0$, and the probability that exactly n packet transmissions are performed $\Pr\{T=n\}$ is given by

$$\Pr\{T=n\}=1-\sum_{j=1}^{n-1}\Pr\{T=j\}=$$
$$=1-\sum_{j=k}^{n-1}\left(\prod_m\left[1-\sum_{i=0}^{k-1}\binom{j}{i}(1-p_m)^i p_m^{j-i}\right]-\prod_m\left[1-\sum_{i=0}^{k-1}\binom{j-1}{i}(1-p_m)^i p_m^{j-1-i}\right]\right).$$

Consequently, the expected number of required broadcast attempts results in

$$E\{T\}=\sum_{t=1}^{n}t\cdot\Pr\{T=t\}$$
$$=\sum_{t=k}^{n-1}t\cdot\left\{\prod_m\left[1-\sum_{i=0}^{k-1}\binom{t}{i}(1-p_m)^i p_m^{t-i}\right]-\prod_m\left[1-\sum_{i=0}^{k-1}\binom{t-1}{i}(1-p_m)^i p_m^{t-1-i}\right]\right\}+$$
$$+n\left\{1-\sum_{j=k}^{n-1}\left(\prod_m\left[1-\sum_{i=0}^{k-1}\binom{j}{i}(1-p_m)^i p_m^{j-i}\right]-\prod_m\left[1-\sum_{i=0}^{k-1}\binom{j-1}{i}(1-p_m)^i p_m^{j-1-i}\right]\right)\right\},$$

which allows specifying the relative throughput $\eta(\mathbf{p})$ of the p-t-m incremental redundancy retransmission scenario exploiting Reed-Solomon codes as

$$\eta(\mathbf{p})=\frac{k}{E\{T\}}.$$

In addition to the expected throughput we are also interested in the residual error rate. Therefore, assume that all n packets corresponding to one RS-code word have already been broadcasted to the MSs. Let X' be a random variable denoting the number of erroneously received packets within the set of n received packets belonging to one code word. Let X be a random variable denoting the number of erroneous packets after RS-decoding. The probability $\Pr\{X'=x'\}$ that exactly x' packets were received in error is given by

$$\Pr\{X'=x'\}=\binom{n}{x'}p_m^{x'}(1-p_m)^{n-x'}.$$

The decoding of the RS decoder fails, if more than n-k packets are received in error. Hence, the probability that exactly x packets are in error after decoding is given by

$$\Pr\{X=x\}=\begin{cases}0 & x\le n-k\\ \Pr\{X'=x\}=\binom{n}{x}p_m^{x}(1-p_m)^{n-x} & x>n-k\end{cases}.$$

Following from this, the expected value $E\{X\}$ for the number of erroneous packets after the RS-decoding is given by

$$E\{X\}=\sum_{x=1}^{n}x\Pr\{X=x\}=\sum_{x=n-k+1}^{n}x\binom{n}{x}p_m^{x}(1-p_m)^{n-x}.$$

Therefore, the residual packet error probability results in

$$p_{res,m}=\frac{E\{X\}}{n}=\frac{1}{n}\sum_{x=n-k+1}^{n}x\binom{n}{x}p_m^{x}(1-p_m)^{n-x}.$$