

## Annex NEW.

### Acceleration of BER tests preserving the statistical relevance

This text contains the following subclauses:

ANEW.1 Properties of the Poisson distribution

Additionally this clause introduces variables and parameters used later on.

ANEW.2 Introduction into the term CONFIDENCE RANGE

Based on a single measurement, a confidence range around this measurement is derived. It has the property that with high probability the final result can be found in this range.

ANEW.3 Application of the confidence range to decide the outcome of the test

The confidence range is compared with the specified BER limit. From the result a diagram is derived containing an early fail and an early pass condition.

ANEW.4 Practical use of the measurement application

As the diagram is not defined over the entire range, proposals are given how to conduct the test, where the diagram is undefined

### ANEW.1 Properties of the Poisson distribution

With a finite number of samples (**ns**), the final bit error ratio **BER** cannot be determined exactly.

Applying a finite **ns**, we measure a number of errors (**ne**).

$ne/ns = ber$  is the preliminary bit error ratio.

In a single test we apply a finite **ns** and we measure a number of errors (**ne**). **ne** is connected with a certain differential probability in the Poisson distribution. We don't know the probability and the position in the distribution conducting just one single test.

Repeating this test infinite times, applying repeatedly the same **ns**, we get the complete Poisson distribution. The average number of errors is **NE**.  $NE/ns$  is the final **BER**.

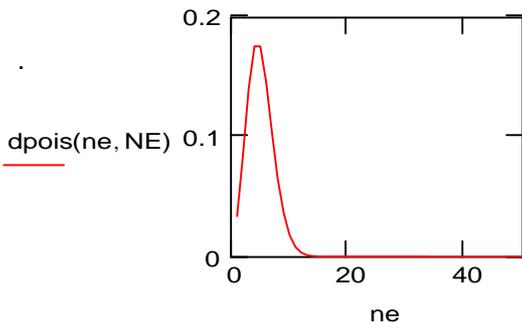
Poisson distribution:  $dpois(ne, NE) = (NE^{ne}/ne!) e^{-NE}$  (1)

$TOL := 10^{-10}$

e.g. :       $ns := 500$            $BER := 0.01$            $NE := ns \cdot BER$

$ne := 1, 2, \dots, 50$

a)



b)

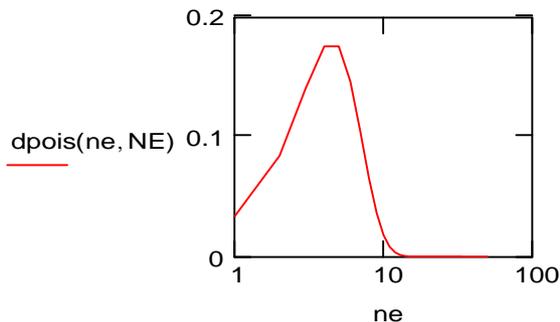


Diagram1

The Poisson distribution has the variable **ne** and is characterised by the parameter **NE**. Real probabilities to find **ne** between two limits are calculated by integrating between such limits.

The width of the Poisson distribution increases proportional to  $SQR(NE)$ , that means, it increases absolutely, but decreases relatively. Increase or decrease **NE** by increasing or decreasing BER and watch the linear scaled (left) and the logarithmic scaled (right) Poisson distribution.

## ANEW.2 Introduction into the term CONFIDENCE RANGE

In a single test we apply **ns** samples and measure **ne** errors. The result can be member of different Poisson distributions each characterized by another parameter **NE**. We ask for two of them:

1)The worst possible distribution **NE<sub>high</sub>** , containing our measured **ne** with [0.2%] probability in the sense

$$0.002 = \int_0^{ne} dpois\_high(ni) dni \quad (2)$$

dpois\_high: the wanted Poisson distribution with the variable ni. (ne is the measured value)

2)The best possible distributions **NE<sub>low</sub>** , containing our measured **ne** with [0.2%] probability in the sense

$$0.002 = \int_{ne}^{\infty} dpois\_low(ni) dni \quad (3)$$

To illustrate the meaning of the range between **NE<sub>low</sub>** and **NE<sub>high</sub>**:

In the case our measured value **ne** is a rather untypical result (just [0.2%] probability) nevertheless the final result **NE** can still be found in this range, called confidence range.

The probabilities **D** in (1) and (2) can be independent like in GSM, but we want to have them dependent and equal.

The inverse cumulative poisson distribution, qpois, answers this question:

Inputs: number of errors **ne**, mesured in this test.

Probabilities **D** and the dual probability **C = 1 - D**

Output: **NE**, the parameter describing the average of the Poisson distribution.

E.g.

Number of errors:  
Confidence level:

$ne := 15$   
 $C := 0.998$

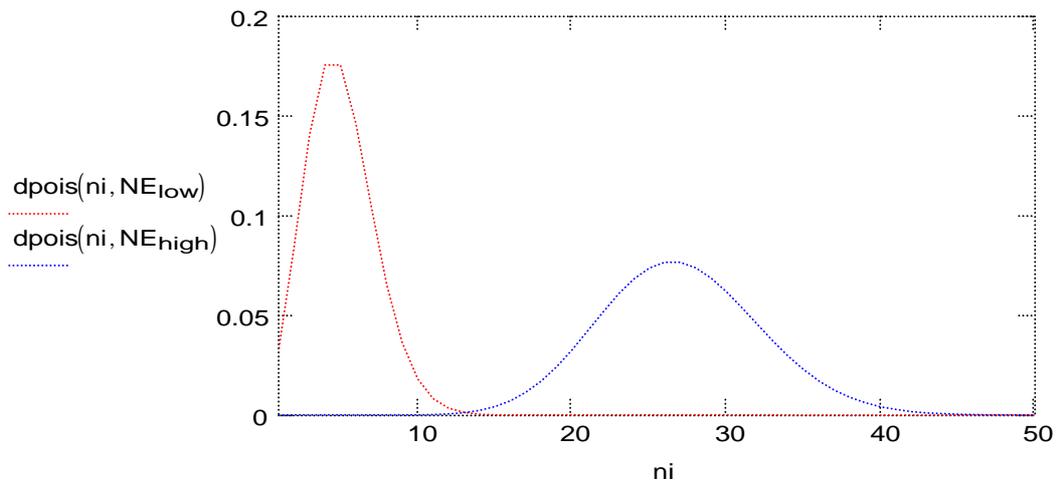
$$D := 1 - C \quad D = 2 \times 10^{-3}$$

$$NE_{low} := qpois(D, ne) \quad (4) \quad NE_{low} = 5$$

$$NE_{high} := qpois(C, ne) \quad (5) \quad NE_{high} = 27$$

$ni := 1, 2.. 50$

a)



b)

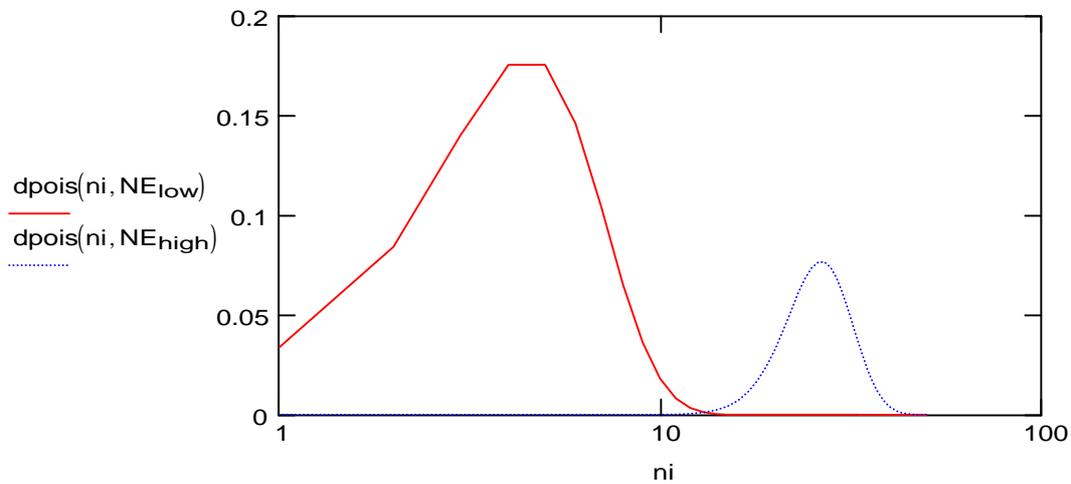


Diagram 2

Same as the width of the Poisson distribution the confidence range increases proportional to  $SQR(ne)$ , that means, it increases absolutely, but decreases relatively.

Increase or decrease  $ne$  and watch the linear scaled (above) and the logarithmic scaled (below) Poisson distributions.  
Set  $C = 0.998$   
Set  $C=0.5$ : the distributions will be congruent.

### ANEW.3 Application of the confidence range to decide the outcome of the test

If we find the entire confidence range, calculated from a single result **ne**, on the good side of the specified limit we can state: With high probability **C**, the final **NE** is better than the limit.

If we find the entire confidence range, calculated from a single result **ne**, on the bad side of the specified limit we can state: With high probability **C**, the final **NE** is worse than the limit.

With each new sample and/or error we consider a new test, reusing all former results. With each new test we update our preliminary data for **ns**, **ne** and **ber**. For each new test we calculate the **confidence range** and check it against the test limit.

Once we find the entire **confidence range** on the good side of the specified limit we allow an early pass.

Once we find the entire **confidence range** on the bad side of the specified limit we allow an early fail.

If we find the confidence range on both sides of the specified limit, it is evident neither to pass nor to fail the DUT early.

Transcription of the above text into formulas:

The current number of samples **ns** is calculated from the preliminary ber and the preliminary ne

$$\mathbf{ber} = \mathbf{ne}/\mathbf{ns} \quad (6)$$

$$\mathbf{BER}_{\mathbf{lim}} = \mathbf{NE}_{\mathbf{limit}} / \mathbf{ns} \quad (7)$$

for abbreviation in the formula we define:  $\mathbf{ber}_{\mathbf{norm}} = \mathbf{ber}/\mathbf{BER}_{\mathbf{limit}} = \mathbf{ne}/\mathbf{NE}_{\mathbf{limit}}$  (normalised ber)

Early pass stipulates:

$$\mathbf{NE}_{\mathbf{high}} < \mathbf{NE}_{\mathbf{limit}} \quad (8)$$

Early fail stipulates:

$$\mathbf{NE}_{\mathbf{low}} > \mathbf{NE}_{\mathbf{limit}} \quad (9)$$

$$C = 0.998$$

$$D = 2 \times 10^{-3}$$

$$\mathbf{ned} := 7, 8.. 1000$$

$$\mathbf{ne} := 1, 2.. 1000$$

The early fail and the early pass limit are displayed in diagram 3:

Due to the fact that the  $NE_{low}$  equals zero for  $ne < 7$ , equation 11 has a singularity The early fail limit is therefore only applied for values of  $ne \geq 7$

Note: To avoid a mathematical error the variable ned in equation 11 starts with ned = 7. Beside this restriction the meaning of ned and ne is the same.

early pass limit:

$$\text{bernorm}(ne, C) := \frac{ne}{\text{qpois}(C, ne)} \quad (10)$$

early fail limit:

$$\text{bernorm}(ned, D) := \frac{ned}{\text{qpois}(D, ned)} \quad (11)$$

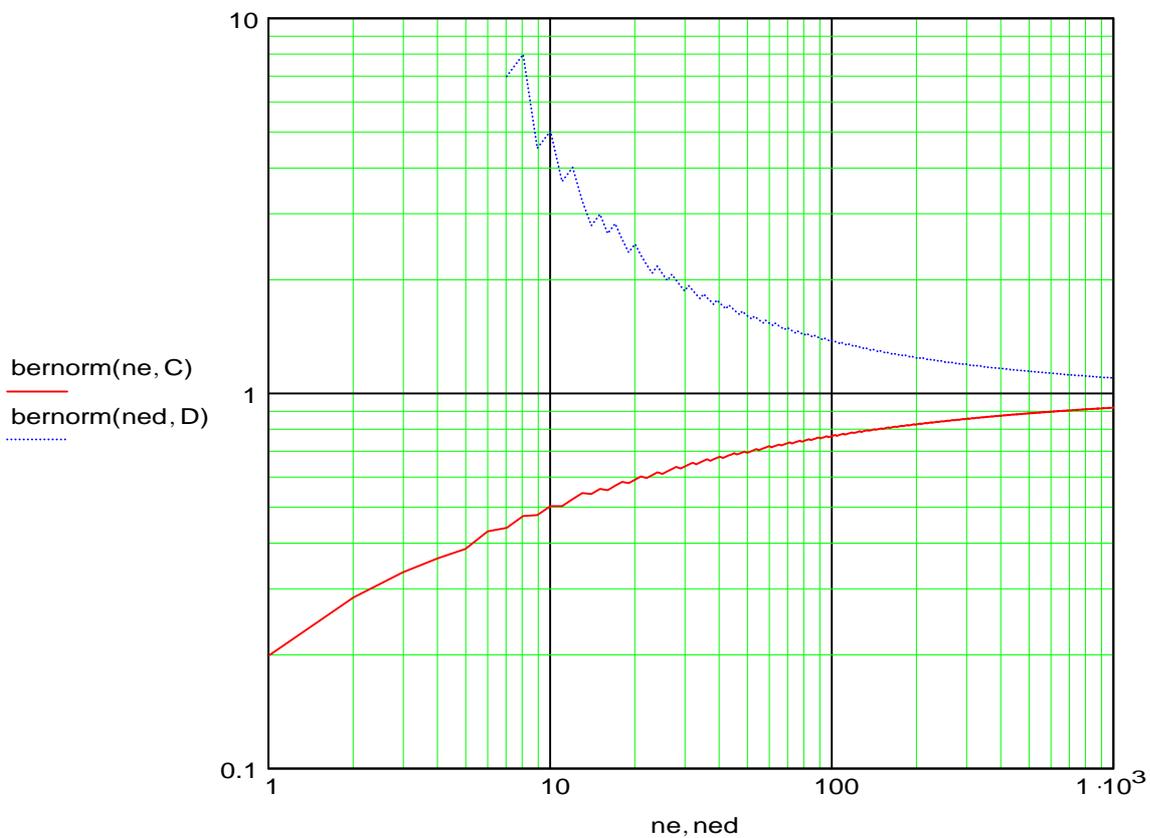


Diagram 3

## ANEW.5 Practical use of the measurement application

### ANEW.5.1 Start

As the early pass limit is not defined for  $ne = 0$  (normally the case at the very beginning of the test for a good DUT), we regard an artificial error event with the first sample. When the first real error event occurs, the artificial error is replaced by this real one.

Note 1: regarding temporarily an artificial error is a conservative approach to use the undefined range of the early pass limit

Note 2: This gives us the shortest possible measurement time for an ideal good DUT:  $ns=5000$  (for  $BER_{limit}=0.001$ , probability 0.2%)

As the early fail limit is not defined for  $ne < 7$  due to singularities, we do not use the early fail limit for any decision until  $[ne=7]$

Note 1: postponing the decision until  $ne=7$  avoids to employ the Poisson distribution in a range where it approximates the binomial distribution not ideally (ber not very low).

Note 2: This ensures that a broken DUT can be found beyond the early fail limit in any case after a few samples, approx 15.

### ANEW.5.2 Early stop

With each new sample and/or error we consider a new test, reusing all former results. With each new test we update our preliminary data for  $ns$ ,  $ne$  and  $ber$  and  $ber_{norm}$  and enter a  $ber_{norm}/ne$  coordinate into the bernorm-diagram. Once the  $ber_{norm}/ne$ -trajectory crosses the early fail limit ( $ber_{norm}(ne,D)$ ) or the early pass limit ( $ber_{norm}(ne,C)$ ) the test may be stopped and the conclusion may be drawn based on this instant.

### ANEW.5.3 Regular stop

If no early stop occurs the BER Test may be stopped, when the following condition is valid:

$$[ \quad ne \geq 200 \quad ]$$

and the DUT shall be passed.

Note 1: Worst case interpretation: With  $C=0.998$ ,  $ne=200$  then  $NE_{high}=242$  and  $NE_{low} = 161$ . The relative distance of  $NE_{high}$  and  $NE_{low}$  is 50%. So a DUT tested against BER 0.0015 (instead of 0.001) is passed and a wrong decision is done with the same low probability 0.2%

Note 2: At this point we can calculate the longest possible measurement time:  $ns=242,000$  (for  $BER_{limit}=0.001$ , probability 0.2%)

### ANEW.5.4 Preparation of the test

The diagram with the early fail limit and the early pass limit can be derived from the probability to pass a bad DUT and to fail a good DUT prior to the test. During the test just the  $ber_{norm}/ne$ -trajectory is entered into the diagram and checked against the above mentioned conditions.